Start-up entry strategies: Employer vs. Nonemployer firms*

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Abstract

From 1997 to 2001 we observe in the USA a faster growth in the number of Nonemployer firms (NF) vis à vis Employer firms (EF). The diverse speed of net entry may be due to particular internal organisation of the two types of firms and the effect that this has on the reactions to market uncertainty. However, the set of internal organizations of firms is larger than that made up simply by EFs and NFs, in particular among newborn firms, since we observe corporate start-ups with employees, firms owned and managed by their founders who are simultaneously the employees and, finally, non corporate enterprises. The second class of firms mostly belongs to the category of

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NFs, according to US nomenclature, while non corporate firms may belong to either category. Our curiosity is attracted by different entry patterns of NFs and EFs and our aim is to interpret them. According to recent literature, firms carry out an irreversible investment, such as entry, only if market prices are strictly larger than average total costs (Marshallian point). However, the trigger price that makes firms become active is affected by institutional rules, the existence of profit sharing, efficiency wages, exit options - i.e. partial reversibility -, financial constraints. Then, the internal organization of a newborn firm may make the difference. In a continuous time stochastic environment, where firms bear a sunk cost, we model entry as a growth option. On the trace of distinct objective functions we show that NFs and EFs have specific entry patterns in terms of output price and/or size. Why? Simply because they react in diverse fashions to market price volatility. In this sense we are able to show that, in most cases, the NF is locally less risky. This makes the NF better suited to enter under conditions of higher volatility. This exactly corresponds to what happened during the years between 1997-2001.

*JEL Classification*: L21, L3, J54, G13:

**Keywords**: Entry Strategies, Uncertainty, Nonemployer, Employer Firms.
1 Introduction

From 1997 to 2001 in the USA we observe a faster growth in the number of Nonemployer firms (NF) vis-à-vis Employer firms (EF). This diverse speed of net entry may at first appear quite odd. However, to a closer scrutiny it seems to be caused by distinct internal organizations that generate different reactions to market uncertainty, quite high in those years of end of millennium.

To validate this statement and better understand the phenomenon considered we have to go through the internal organization of firms according to the category they belong and their infant history.

Start-up firms (SUFs) are the most dynamic part of the economy with their active development of new goods and technological endeavors. Yet, most of their operating modes are heterogeneous with respect to incumbent consolidated firms. Often their actions do not adhere to the traditional market canons of Marshallian enterprise and their internal organization departs, in many respects, from that of a purely profit-value maximizing firm (PMF). The inner structure and the governance of SUFs show a large variety of organization modes. Most of them appear to be quite far from those pertaining either to the publicly owned corporate, run on behalf of shareholders, or to the private corporate, whose control is in the hands of a family or an individual owner. By limiting the variety of organizational forms, we may distinguish at least two kinds of SUFs.

The first corresponds to newborn firms made up of few people who are simultaneously the owners and the employees of the venture they have created.

\footnote{By the US definition of corporate we mean a firm with limited liability of the owners. According to UK nomenclature this firm is either a public limited company (PLC) or a private limited company (LTD).}
ated. This kind of firm mostly belongs to the *Nonemployer* (*NF*) category comprising enterprises of three distinct legal and/or organizational forms: *Individual Proprietorship*,\(^2\) *Partnerships, Corporations*, all without employees. (according to the US Bureau of Census nomenclature; US Census Bureau, 2003a). The most common are the first two.\(^3\)

The *second* kind of *SUF* belongs to the traditional *Employer* (*EF*) category, whose governance replicates that of a *PMF*, with separation between employers and shareholders.

*SUFs* quickly grow or disappear. In the first case they often undergo thorough transformation. Some become public or private *corporate* after an initial period of *noncorporate*.\(^4\) Others are taken over by consolidated firms. Sometimes the transformation is more radical than what the pure change of the legal status may hint. But before undergoing dimensional, financial, legal

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\(^2\)These firms are close to the *self-employed* category of the European nomenclature. See for instance Parker, Barnby and Belghitar (2004).

\(^3\)Here follows the US Bureau of Census definition: “*Individual proprietorship*...is an unincorporated business owned by an individual”. Self-employed persons are included in this category. “*Partnership* .....is an unincorporated business owned by two or more persons having a shared financial interest in the business”, i.e. sharing profits and losses and responsibilities having a general or limited liability. “*NF Corporation* is a legally incorporated business under state laws”, without employees. See: http://www.census.gov/epcd/nonemployer/view/define.html

\(^4\)See Steingold (1999). In addition see the advice of a large Bank like NatWest of UK to *SUFs* as to the choice of their Legal status, with a particular emphasis on the Unincorporated Partnership mode of organization.

http://www.natwest.com/smallbusiness/
guides/startingup/
index.asp?navid=SBS/
FINANCIAL_GUIDES/STARTING_UP/
LEGAL_STATUS&pid=2
and governance metamorphosis, turning a baby firm into a mature corporate enterprise, a SUF is a strange animal whose behavior may be at odds with standard modelization of PMFs. Why? And in which sense?

The answer may come from the very existence of a large bulk of NFs and in particular Partnerships. If we confine to the internal organization of the newborn NFs, we may find that Partnerships closely replicate Labor Managed firms (LMF) and individual Proprietorship resemble LMFs, even though in the limit due the single member structure. A similar proposition may be stated for NF Corporations. In LMFs, owners and employees coincide while sharing the governing power of the firm on an equal foot. Surprisingly enough, this is something that can be found in most NFs, whose market behavior should then be expected to replicate fairly closely that of a LMF. The question then boils down to what are the implications for entry strategies of this odd similarity between NFs, one of the most dynamic form of modern baby production, and LMFs, that most analysts regard as a sort of bulky legacy of socialism.  

Here, we analyze entry strategies of firms belonging either to the NFs

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5See the canonical models of Ward (1958), Vanek (1970), the refinements of Pestieau and Thisse (1979), the empirical scrutiny of Pencavel and Craig (1994), the analysis of LMF oligopolies in Delbono and Rossini (1992), workers’ enterprises in Sertel (1993), just to mention few contributions.

6The theory of the LMF has evolved quite a lot in parallel with the growing success of market economies vis à socialist economies. Literature has shown the long run affinities between a competitive LMF and its corresponding PMF, despite heterogeneous behavior in the short run. However, some of the problems concerning both the perverse response of the short run supply and the difficulty of LMFs to get credit are assumed away once we introduce tradeability of memberships - i.e.: workers’ enterprises (Sertel, 1982; 1997) - or we assume that credit provided by members of the LMF is subordinate with respect to the credit by banks or by any third agent (Jossa and Cuomo, 1997).
or to the EFs. Our setting is a dynamic and uncertain one, where a new venture is defined as a project that can be carried out at different points of time and at different entry-trigger market prices.

Most differences among the two kinds of firms come from the existence of uncertainty coupled to sunk costs. Thanks to the proximity between the NF and the LMF we show that, in an uncertain dynamic environment, there are circumstances where the NF enters at less favorable conditions becoming the swiftest start-up, while in other circumstances the EF is smarter. Moreover, we analyze the entry strategies and the size of firms and interpret the recent growth of NFs in the US during a period of intense financial volatility. Our aim is to see how market price volatility may favor one particular firm organization.

A by-product of this investigation is that the entry trigger price increases in distinct fashions for the two kinds of firms. A larger variance makes the investment return more volatile. The value of the option grows but there is a larger incentive to delay entry\(^7\). In the NF each member shares this option with colleagues. Therefore, he has to bear only a fraction of the entry cost. If so, the outcome is a higher value of the option without any increase in the incentive to delay entry.

In the next section some data about NFs and EFs are shown. In the third section we present the basic set up. In section four we investigate different entry policies. In the fifth some comparisons are carried out. In the final section concluding remarks are drawn.

\(^7\)This effect follows from the "bad news principle of irreversible investment" (Bernanke, 1983).
2 Employer and Nonemployer businesses in the US

When considering size we soon discover that many firms are very small and often made up just by the proprietor (Individual Proprietorship) or by few guys who own the firm in a Partnership mode. US Census data say that the number of these firms belonging to the NF category is rather high. Look at Table 1. The establishments (est) of NFs are more than twice those of EFs. Between the Censuses of 1997 and 2001 the number of NFs grew by 10%, while EFs just by 3%. If establishments are a proxy of the number of firms and NFs do not live, on average, longer than EFs\textsuperscript{8}, we may conclude that, during the period 1997-2001, the entry of NFs is more likely and easier than that of EFs. Nonetheless, the weight of NFs in terms of the share of income produced is lower, as we can see by comparing the receipts (RE) of NFs and the payrolls (PA) of EFs\textsuperscript{9}. This proves that usually the NFs are much smaller than the corresponding EFs.

\textsuperscript{8}See Parker (2004) and Taylor (1999).
\textsuperscript{9}Of course these two magnitudes are quite heterogeneous, yet we compare them just qualitatively, without any measuring purpose.
### TABLE 1

Employer (EF), Nonemployer (NF) businesses in all US industries\(^{10},^{11}\)

<table>
<thead>
<tr>
<th>Year</th>
<th>NF</th>
<th>Δ</th>
<th>EF</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>est</td>
<td>15,438,609</td>
<td>w: 10%</td>
<td>est</td>
</tr>
<tr>
<td>1998</td>
<td></td>
<td>15,708,727</td>
<td>1.7%</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td></td>
<td>16,152,604</td>
<td>2.8%</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>16,529,955</td>
<td>2.3%</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td></td>
<td>16,979,498</td>
<td>2.7%</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>re</td>
<td>586,315,757</td>
<td>w: 24%</td>
<td>pa</td>
</tr>
<tr>
<td>1998</td>
<td></td>
<td>643,720,460</td>
<td>9.7%</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td></td>
<td>667,219,733</td>
<td>3.7%</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>709,378,836</td>
<td>6.3%</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td></td>
<td>729,922,063</td>
<td>2.8%</td>
<td></td>
</tr>
</tbody>
</table>

From Table 1 we calculate in 2001 the average receipts (re) for the NF that was 43,000 dollars while the average payroll (pa) for the EF was 442,000 dollars. The two magnitudes (receipts and payrolls) provide proxy measures of relative size of the two categories of firms. As for the percentage variation (Δ) over the entire period (w), for NFs is larger when considering the number of establishments (10% versus 3%), rather than receipts and payrolls

\(^{10}\)Payrolls (pa) and receipts (re) are in thousands of current dollars.

\(^{11}\)\(w\) means percentage variation on the whole period 2001/1997. For NFs we have only receipts. For EFs we have payrolls in thousands of current dollars.

\(^{12}\)2002 figure is 17,646,062. and variation (w) over the period 1997-02 is 14.3%.

\(^{13}\)2002 figure is 770,032,328 and variation over the period (w) 1997-02 is 31.3%.
(24% versus 30%). Over the same period the number of establishments of *Partnerships* increased by 26%, the largest rate of growth among all categories. The average size of establishment in 2001 is 123,000 dollars, larger than the average of NFs, but still lower than EFs. Receipts of *Partnerships* increased over the same time span by 39%.

3 The basic set up

Here is the basic framework drawing the borders of the environment where we wish to compare the behavior of two *SUFs*: a *NF* and an *EF*.

We assume that:

1) Firms undertake a project of finite size, corresponding to the start-up decision. We consider a firm in isolation, even though there are scanty differences with respect to a competitive market (Leahy, 1993).

2) The investment is irreversibly sunk. It can neither be changed, nor temporarily stopped, nor shut down. Other operating options are neglected for the sake of simplicity of comparisons\(^{14}\). The commitment is equal to *K*.

3) When the firm is operative the instantaneous short run revenue is

\[
R(p_t; L_t) \equiv p_t Q(L_t)
\]

\(^{14}\)This avoids the analysis of operating options which differ between the two kinds of firms. The most relevant one is due to the ability of the firm to reduce output or even shut down, thereby eschewing variable costs. Operating options increase the value of the firm. See, among others, MacDonald and Siegel (1986) and, for a thorough discussion, Dixit and Pindyck (1994, chs. 6 and 7).
where $p_t$ is the market output price, $L_t$ is labor input, $Q(L_t)$ is the short run Marshallian production function, with $Q(0) = 0$, $Q'(L_t) > 0$, $Q''(L_t) < 0$.

4) The market price is uncertain and evolves over time according to the following trendless stochastic differential equation:

$$dp_t = \sigma p_t dB_t$$

with $\sigma > 0$ and $p_0 = p$, (2)

where $dB_t$ is the increment of a standard Wiener process and the volatility parameter ($\sigma$) is constant over time.

5) The market wage for unit of labor $w$ is constant.

6) The investment is financed either by founding employee-members, in the case of the NF,\textsuperscript{15}, or by shareholders, in the case of the EF.

### 3.1 The Nonemployer Firm (NF)

We investigate the NF decision to start-up a new venture and assume that:

(i) the number of employees-members is held fixed after entry\textsuperscript{16}; (ii) each member investing in the project maximizes the discounted value of expected individual net dividends; (iii) each member receives the sum of the accounting dividends plus the contractual wage $w$.

We solve this problem backwards. First, for any size of the NF (level of $L$) we evaluate the member option value to enter. Subsequently, we choose $L$ that maximizes the individual option value.

\textsuperscript{15}This is consistent with the assumption of the existence of a market for NF memberships, operating according to standard financial canons (Sertel, 1993, 1997).

\textsuperscript{16}In this sense the NF faces a kind of constraint since it cannot change $L$ and it must decide the optimal entry time.
To evaluate the individual option we calculate the discounted value of expected net individual dividend:

\[
y(p; L) - \frac{w}{\rho} = \frac{E_0 \left\{ \int_0^\infty e^{-\rho t} R(p_t; L) dt \mid p_0 = p \right\}}{L} - K - \frac{w}{\rho} \tag{3}
\]

where \(E_0(.)\) is the expectation operator, with the information available at time zero, \(\rho\) the riskless interest rate,\(^{17}\) and \(\frac{w}{\rho}\) the discounted flow of the market wage, which corresponds to the minimum the \(NF\) grants its employee-members, i.e., a participation constraint.

The employee-member belonging to a \(NF\) of size \(L\) decides whether and when to start the project by solving an optimal stopping time problem, i.e. choosing the investment timing which maximizes:

\[
f_{NF}(p; L) = \max_T E_0 \left[ \left( y(p_T; L) - \frac{w}{\rho} \right) e^{-\rho T} \mid p_0 = p \right] \tag{4}
\]

Members of the \(NF\) are homogeneous. Each one holds an option to invest corresponding to (4) and has an interest to exercise cooperatively its option at the same time.\(^{18}\) The option value comes from solving (4). The value of (4), prior to investment, (see McDonald and Siegel, 1986; Dixit and Pindyck, 1994), is:

\[
f_{NF}(p; L) = \left( \frac{p_{NF} Q(L)}{\rho L} - \frac{w}{\rho} - K \right) \left( \frac{p}{p_{NF}} \right)^\beta \text{ for } p < p_{NF}(L) \tag{5}
\]

\(^{17}\)Introducing risk aversion does not change the results since the analysis can be developed under a risk neutral probability measure (Cox and Ross, 1976; Harrison and Kreps, 1979).

\(^{18}\)Members have just founded the firm of the optimal size and they have no incentive to behave noncooperatively from the beginning.
where \(1 < \beta < \infty\) is the positive root of the auxiliary quadratic equation

\[\Psi(\beta) = \frac{1}{2}\sigma^2 \beta(\beta - 1) - \rho = 0\]

and \(P_{NF}\) is the critical price that makes the \(Lth\) employee-member indifferent between investing right away or waiting. Maximizing (5) with respect to \(P_{NF}\) we obtain the candidate policy for optimal NF start-up as:

\[\text{Lemma 1} \quad \text{The employee-member’s optimal strategy requires investing as soon as the market price exceeds the break-even threshold } P_{NF}, \text{ where:}\]

\[P_{NF}(L) = \frac{\beta}{\beta - 1} \rho AC(L) \quad \text{with} \quad \frac{\beta}{\beta - 1} > 1 \tag{6}\]

where \(AC(L) = \frac{wL + K}{Q(L)}\) is long-run average total cost.

Substituting (6) into (5) and rearranging we write, in reduced form, the \(Lth\) employee-member’s value of the project prior to invest:

\[f_{NF}(p; L) = A(L) p^\beta \quad \text{for } p < P_{NF}(L), \tag{7}\]

where the constant \(A(L)\) is given by:

\[A(L) = \frac{(\beta - 1)^{\beta-1}}{(\rho \beta)^{\beta}} a(L) \tag{8}\]

\[= \frac{(\beta - 1)^{\beta-1}}{(\rho \beta)^{\beta}} [Q(L)]^\beta \frac{[wL + K]}{L^{\frac{\beta}{\rho} L + K}]^{\beta-1}} > 0\]

What is the optimal NF dimension? By (7) the optimum requires choosing the \(L\) for which \(A(L)\) is the largest. Moreover, by (8), the optimum dimension maximizes \(a(L)\), which yields the first order condition (FOC):\(^{19}\)

\[^{19}\text{Taking logs of } a(L) \text{ we have } \beta \log Q(L) - (\beta - 1) \log (\frac{w}{\rho} L + K) - \log L, \text{ or equivalently} \log Q(L) - (\beta - 1) \log (\frac{w}{\rho} L + K) - \frac{1}{\rho} \log L.\]
Lemma 2 The optimal size of the NF can be obtained from:

\[ \frac{L_{NF}Q'(L_{NF})}{Q(L_{NF})} = 1 - \frac{(\beta - 1)}{\beta} \frac{K}{\frac{\nu}{\rho}L_{NF} + K} \]  

(9)

Since the r.h.s. of (9) is less than one, a necessary condition for an optimal solution is an output elasticity \( \varepsilon_{QL} \equiv \frac{LQ'(L)}{Q(L)} < 1 \), i.e. if the average productivity \( \frac{Q(L)}{L} \) is a decreasing function of labor (as from Assumption 3).

Although we cannot prove the second order condition (SOC) on a general basis, we can provide examples of functions that are locally concave, such as the following calibrated version of \( a(L) \), i.e.: \( Q = \log \lambda L \). We mostly follow suggestions from other studies (Dixit and Pindyck, 1994) and calibrate \( \rho = 0.04 \), \( \sigma = 0.2 \), \( w = 3 \), \( \lambda = 2.7 \), and \( K = 10 \). Figure 1 below shows a local maximum, the interior solution, that occurs between \( L = 1 \) and \( 2 \) at \( L = 1.07 \).
3.2 The Employer Firm (EF)

When does an EF enter the market? Entry is a project of infinite life and the firm has to properly tune the input $L$ over time. To ease comparisons we make a simplifying hypothesis that parallels assumption (i) on the NF: labor is variable only ex-ante. The EF selects its project among a set of ventures with the same $K$, but distinct levels of labor.\footnote{This means a putty-clay technology.} The market value of a project of dimension $L$ turns out to be:

$$V(p) = E \left\{ \int_0^\infty e^{-pt} (R(p_t, L) - wL) \, dt \right\} = \left( \frac{pQ(L)}{\rho} - \frac{wL}{\rho} \right).$$
Whether and when to ignite the project is the solution of a standard optimal stopping time problem:

\[ F_{EF}(p; L) = \max_T E_0 \left[ \left( \frac{pQ(L)}{\rho} - \frac{wL}{\rho} - K \right) e^{-\rho T} | p_0 = p \right] \quad (10) \]

Owing to the homogeneity of (3), we have:

\[ F_{EF}(p; L) = f_{NF}(p; L) L \quad (11) \]

where \( f_{NF}(p; L) \) is the value of the project for the \( L \)th member of the \( NF \), given by (4). By analogy with (5), and, as a result of the above arguments, we obtain:

**Lemma 3** The \( EF \) optimal strategy dictates investing as soon as the price exceeds the break-even level \( p_{EF} \), where:

\[ p_{EF}(L) \equiv \frac{\beta}{\beta - 1} \rho AC(L) \quad \text{with} \quad \frac{\beta}{\beta - 1} > 1. \quad (12) \]

Then, substituting (12) into (11), the value of the option for an \( EF \) is:

\[ F_{EF}(p; L) = B(L)p^\beta, \quad \text{for} \quad p < p_{EF}(L) \quad (13) \]

where the constant \( B(L) = LA(L) \) is:

\[ B(L) \equiv \frac{(\beta - 1)^{\beta - 1}}{(\rho\beta)^\beta} b(L) \quad (14) \]

\[ = \frac{(\beta - 1)^{\beta - 1}}{(\rho\beta)^\beta} \frac{[Q(L)]^\beta}{[\frac{wL}{\rho} + K]^{\beta - 1}} > 0 \]

Again, by (13), the optimum requires finding the \( L \) for which \( B(L) \) is the largest. Moreover, by (14), efficient size maximizes \( b(L) \) yielding the following FOC:

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21 This framework is similar to that of Dixit (1993), even though here we consider a continuum of projects with total costs, \( \frac{w}{\rho}L + K \), which are linear in the labor input.

22 Taking logs of \( b(L) \) we get \( \beta \log Q(L) - (\beta - 1) \log (\frac{wL}{\rho} + K) \), or
Lemma 4 \textit{The optimal dimension of the EF is given by:}

\begin{equation}
\frac{L_{EF}Q'(L_{EF})}{Q(L_{EF})} = \frac{(\beta - 1)}{\beta} \left( 1 - \frac{K}{(\frac{w}{p}L_{EF} + K)} \right)
\end{equation}

As for the NF, since the r.h.s. of (15) is less than one, a necessary condition is a production elasticity $\varepsilon_{QL} < 1$.

Going through the same calibration of the NF, we get a function $B(L)$ depicted in Figure 2.

\textbf{Figure 2 :} Function $B(L)$ - EF case

$\log Q(L) = \frac{(\beta - 1)}{\beta} \log \left( \frac{wL}{p} + K \right)$.

\footnote{It goes without saying that if entry costs are null, condition (15) reduces to: $\frac{LQ'(L)}{Q(L)} = \frac{(\beta - 1)}{\beta} < 1$. This is equivalent to the condition proposed by Dixit (1993) for an EF firm that has to choose among alternative investment projects of different dimension. See also Moretto (2003) for an analogous condition for an EF firm that can incrementally contract its capacity.}
A maximum for $L$ lies between 2 and 3. If we compare the two firms, we see that the optimal size of the $EF$ is 2.99 ($\approx 3$)\textsuperscript{24} while for the $NF$ is 1.06 ($\approx 1$).\textsuperscript{25}

4 NF versus EF entry strategies

On the basis of Lemma 2 and 4, we are able to show that:

**Proposition 1** a) Over the range where the second order condition holds, the $EF$ is operating with a higher dimension than the $NF$, i.e.:

$$L_{NF} < \hat{L} < L_{EF},$$

where $\hat{L} = \text{arg min } AC(L)$ is the minimum efficient scale.\textsuperscript{26}

b) The entry trigger prices react in distinct ways according to the firm’s organization, i.e.:

$$\frac{\partial p_{EF}}{\partial L_{EF}} > 0 \quad \frac{\partial p_{NF}}{\partial L_{NF}} < 0.$$

**Proof.** See Appendix.

The first part of the proposition confirms the above numerical calculations shown in Figure 1 and 2 and it is consistent with the empirical finding that $NF$s are smaller than corresponding $EF$s.\textsuperscript{27}

\textsuperscript{24}The number in brackets is the closest integer, since we do not use integer programming.
\textsuperscript{25}The two triggers $p_{NF}(L)$ and $p_{EF}(L)$ are, respectively, 63.11 and 44.83.
\textsuperscript{26}With the above calibration $\hat{L} = 1.13$.
\textsuperscript{27}This is also consistent with literature on LMFs. " Labor-managed firms will be smaller than their capitalist counterparts in the short-run when profit are positive" (Bonin and Putterman, 1987, p.15). The same applies to the long run if profits are strictly positive (ibidem, p.57).
To appreciate the intuition behind this result we go back to Lemmas 1-4 and rewrite the FOCs for the optimal dimension (9) and (15) at entry. For the $EF$, by multiplying both sides of (15) by $p_{EF}(L_{EF})$ and simplifying we get:

$$p_{EF}(L_{EF})Q'(L_{EF}) = w$$  \hspace{1cm} (16)

Then, the $EF$, at entry, decides the optimal dimension equating the value marginal product to the market wage $w$. The $EF$ is using $L$ efficiently.

By analogous procedure for the $NF$ we obtain:

$$p_{NF}(L_{NF})Q'(L_{NF}) = w + \frac{1}{\beta - 1}(w + \rho \frac{K}{L_{NF}}) > w.$$  \hspace{1cm} (17)

Unlike the $EF$, the $NF$ chooses the optimal size equating the value marginal product to the full wage, which exceeds the market wage $w$. The Marshallian full cost of the investment imputed to each employee-member is $w + \rho \frac{K}{L_{NF}}$, larger than $w$ since members of the $NF$ possess an option (to delay entry), not owned by employees of an $EF$. Would-be employee-members are special workers endowed with an option to give rise to a kind of Partnership and are therefore “more skillful” deserving a compensation larger than $w$. By the decreasing marginal product of labor, at entry the $NF$ will have a smaller size than its twin mate $EF$, i.e. $L_{NF} < L_{EF}$.

The conclusion that the $NF$ and the $EF$ have different dimensions opens the way to further questions about the entry price as size changes. However, as the second part of Proposition 1 suggests, we cannot tell which one enters first, even though the $NF$ and the $EF$ operate at different scales ($NF$ to the left of the minimum efficient scale $\hat{L}$, while $EF$ to the right).

Both the $NF$ and the $EF$ undertake the entry investment when the market price equals the average total cost $AC(L) \equiv \frac{wL + K}{Q(L)}$ multiplied by a
coefficient $\frac{\beta}{\beta - 1} \rho$. Then, by the U-shaped $AC(L)$ function we may observe, at
the same market price, small $NF$s and larger $EF$s entering the market, as
data show in section 2.

4.1 A local study of the entry strategies

For more fleshy intuition on entry triggers we do some comparative statics on the effect of uncertainty. The first important result is:

**Proposition 2** If $\sigma = 0$ the $EF$ and $NF$ operate at the minimum efficient scale, i.e.:

$$L_{NF} = \hat{L} = L_{EF}$$

and the entry strategy is the same, i.e.:

$$p_{EF}(\hat{L}) = p_{NF}(\hat{L}).$$

**Proof.** See Appendix.

If $\sigma \to 0$, uncertainty disappears, $\beta = +\infty$ and $\frac{\beta - 1}{\beta} = 1$. As volatil-
ity vanishes, entry sizes converge to the minimum efficient scale and entry
strategies coincide.\(^{28}\)

A second result is:

**Proposition 3** As market price volatility increases the entry price increases for both firms:

$$\frac{\partial p_{NF}}{\partial \sigma} > 0 \text{ and } \frac{\partial p_{EF}}{\partial \sigma} > 0.$$

while the gap between sizes widens, i.e.

$$\frac{\partial (L_{EF} - L_{NF})}{\partial \sigma} > 0$$

---

\(^{28}\)If the market price has a positive drift $\alpha > 0$, then $\beta = \rho/\alpha$ and $\frac{\beta - 1}{\beta} = \frac{\rho - \alpha}{\rho}$. Therefore the deterministic results conform to those of the uncertainty case.
Proof. See Appendix.

As the real option theory predicts, we show that increasing risk puts off investment timing, i.e. the entry price increases with uncertainty. This follows from the "bad news principle of irreversible investment". A larger market variance makes the investment return more volatile with positive effect on the option to invest. However, the net marginal benefit of waiting, arising from the avoidance of an investment in the bad state, increases with uncertainty. This induces an entry delay (Bernanke, 1983).

<table>
<thead>
<tr>
<th>$\rho = 0.04$</th>
<th>$L_{EF}$; $p_{EF}$</th>
<th>$L_{NF}$; $p_{NF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.00$</td>
<td>$(\bar{L}) 1.13$; $p(\bar{L}) 3.40$</td>
<td>$(\bar{L}) 1.13$; $p(\bar{L}) 3.40$</td>
</tr>
<tr>
<td>$\sigma = 0.08$</td>
<td>1.57; 9.84</td>
<td>1.10; 5.36</td>
</tr>
<tr>
<td>$\sigma = 0.20$</td>
<td>2.99; 39.14</td>
<td>1.07; 7.66</td>
</tr>
<tr>
<td>$\sigma = 0.33$</td>
<td>7.83; 218.67</td>
<td>1.05; 11.10</td>
</tr>
</tbody>
</table>

Furthermore, as uncertainty increases, the $NF$ gets larger and the $EF$ smaller, making for a wider gap.

The intuition behind such a result may be better grasped referring to conditions (16) and (17). As usual, for the $EF$, the higher entry price makes the firm react by increasing the optimal size so as to keep the value marginal product in pace with the market wage.

For the $NF$, the full wage imputed to each employee-member goes up with $\sigma$, and the firm may desire to reduce its size to adjust the value marginal product. The calibrated comparison is in Table 3.
Proposition 3 states the impossibility of a global rank in terms of entry prices for the two distinct firms. Nonetheless, for small values of price volatility the above asymmetry of behavior benefits the NF. In particular, by Proposition 2, we may derive the following:

**Corollary 1** *In terms of entry strategies, the NF is locally less risky than the EF.*

**Proof.** See Appendix.

The most striking result springs from local analysis around \( \sigma = 0 \). For low price volatility, the NF is locally less risky than the EF, since the NF’s set of entry prices is “less convex” than that of the EF. As it can be seen in Figure 3, the entry boundary increases in different fashions for the two kinds of firms.

Since the employee-members of a NF share equally the option to invest, they may demand a higher reward and require a smaller dimension to compensate for the increased risk. This lowers the net marginal benefit of waiting of each individual member, reducing the incentive to delay entry.
4.2 Discussion

Proposition 3 maintains that the lower is the volatility of the market price, the narrower is the gap between price and average total cost required to make the irreversible investment to enter. The vanishing of uncertainty makes the size of the two firms converge to a unique level, one from above (the EF) and the other from below (the NF) since one increases it size with uncertainty, while the other does the opposite. In the deterministic case, EFs and NFs perform the same way and share the same optimal entry strategy (Proposition 2).

The employee-members receive a “salary” that is the sum of $w$ plus the
option to invest, whose value grows with uncertainty and makes entry strategies diverge. In the case of the \( EF \) the option is held by shareholders. While, the option to start-up in the hands of employee-members reflects their skill to set up a firm. This difference is the one that commands a higher reward as uncertainty and sunk costs enter the picture. All these considerations are independent of the market structure in which the \( SUF \) is embedded (see Leahy, 1993).

From the three Propositions the \( NF \) appears a more suitable entrepreneurial organization in times of high volatility, as the 1997 - 2001 period was. After all, as shown in Pastor and Veronesi (2004), volatility boosts the value of a firm even if there is no bubble.

5 Conclusions

We have gone through entry policies of two kinds of firms, \( EFs \) and \( NFs \), to partially explain why, during the years between 1997 and 2001, we have observed a faster growth in the number of \( NFs \) vis à vis \( EFs \).

We have seen that the main differences between the two firms are their size at entry and their way to react to uncertainty. The \( NF \) enters at a smaller size while the \( EF \) at a larger size. Moreover the \( EF \) is more risky around the entry price than the \( NF \). Both statements may explain:

1. why there are so many entries of \( NFs \) during a period of high volatility such as the years between 97-2001
2. the smaller operation scale of \( NFs \).

Evidence coming from US Census data is definitely consistent with the theoretical observation that \( NFs \) are smaller than \( EFs \).

The divergence between the two entry policies is due to the irreversible
commitment that is associated with entry decision under uncertainty.

Employee-members of the NF hold an option to enter whose value increases with market volatility. The option adds to the market wage making the total "salary" paid higher with respect to the EF. Under decreasing marginal productivity of labor, the NF enters at a smaller scale whenever there is price volatility. Employee-members of a NF hold an option that is the signal of a special ability to set up a firm of their own. The option value increases with uncertainty and the size of the irreversible commitment, making the reward for employee-members differ from the sheer market wage even in the long run equilibrium of the firm. All these considerations are consistent with the conclusion that the NF is less risky.

6 Appendix

6.1 Proof of Proposition 1

Let start proving the first part of the proposition. To do this let's us recall that the EF's optimal dimension is given by:

$$\max_L b(L) = \max_L La(L).$$

The FOC is:

$$b'(L) = a(L) + La'(L) = 0,$$

while the SOC is:

$$b''(L) = a'(L) + a'(L) + La''(L) = 2a'(L) + La''(L) < 0.$$

In general $a''(L) < 0$ does not imply that $b''(L) < 0$: the two regions, where the SOC holds, overlap only partially. Therefore, we confine to their overlap-
ping set. That is, over the range where the SOC holds, since \( L_{NF} \) is such that \( a'(L_{NF}) = 0 \), we have that \( b'(L_{NF}) = a(L_{NF}) > 0 \). Then, if there exists a \( L_{EF} \) such that \( b'(L_{EF}) = 0 \), this will necessarily be \( L_{NF} < L_{EF} \).

For the second part let’s define the average cost function \( AC(L) \equiv \frac{\bar{z}L + K}{Q(L)} \). By the concavity of \( Q(L) \) it is easy to show that \( \lim_{L \to 0} AC(L) = +\infty \) and \( \lim_{L \to +\infty} AC(L) = +\infty \). Further, taking the derivative with respect to \( L \), we get:

\[
\frac{\partial AC}{\partial L} = \frac{\bar{w}Q(L) - (\bar{w}L + K)Q'(L)}{Q(L)^2},
\]

or:

\[
\frac{\partial AC}{\partial L} = \begin{cases} 
< 0 & \text{if } \varepsilon_{QL} = \frac{LQ'(L)}{Q(L)} > 1 - \frac{K}{(\bar{w}L + K)} \\
> 0 & \text{if } \varepsilon_{QL} = \frac{LQ'(L)}{Q(L)} < 1 - \frac{K}{(\bar{w}L + K)}
\end{cases}
\]  

(18)

So there exists a value \( \hat{L} > 0 \) such that \( \frac{\partial AC}{\partial L} = 0 \). This is given by:

\[
\frac{\hat{L}Q'(\hat{L})}{Q(\hat{L})} = \left( 1 - \frac{K}{(\bar{w}\hat{L} + K)} \right).
\]

(19)

The second order derivative is:

\[
\frac{\partial^2 AC}{\partial L^2}(\hat{L}) = -\frac{\bar{w}}{\rho}(\hat{L} + K)Q''(\hat{L}) > 0,
\]

which confirms that \( AC(L) \) is a convex function with a minimum represented by \( \hat{L} \).

Since \( \frac{(\beta - 1)}{\beta} < 1 \), from the comparison between (19) and (15), we have that:

\[
\frac{(\beta - 1)}{\beta} \left( 1 - \frac{K}{(\bar{w}\hat{L} + K)} \right) < 1 - \frac{K}{(\bar{w}\hat{L} + K)},
\]

which, in the range where the SOC holds, implies that \( \hat{L} < L_{EF} \).
On the contrary, from the comparison between (19) and (9), we notice that the \( NF \) operates only in the descending branch of the curve to the left of the minimum. That is, we get:

\[
1 - \frac{(\beta - 1)}{\beta} \frac{K}{(\frac{w}{\rho}L + K)} > 1 - \frac{K}{(\frac{w}{\rho}L + K)}
\]

\[
-\frac{(\beta - 1)}{\beta} \frac{K}{(\frac{w}{\rho}L + K)} + \frac{K}{(\frac{w}{\rho}L + K)} > 0
\]

\[
(1 - \frac{(\beta - 1)}{\beta}) \frac{K}{(\frac{w}{\rho}L + K)} > 0
\]

\[
\frac{1}{\beta} \frac{K}{(\frac{w}{\rho}L + K)} > 0,
\]

which implies that \( \hat{L} > L_{NF} \). Then, the second part follows by convexity of \( AC(L) \) around \( \hat{L} \).

QED

6.2 Proof of proposition 2

If \( \sigma \rightarrow 0 \) we get \( \beta \rightarrow +\infty \) and \( \frac{\beta - 1}{\beta} \rightarrow 1 \). By direct inspections of (16) and (17) (or equivalently (9) and (15)), we get the first part.

6.3 Proof of proposition 3

By applying the implicit function theorem to (15) and (9), it can be shown that \( \partial L_{EF}/\partial \beta \leq 0 \leq \partial L_{NF}/\partial \beta \). Then, since \( \frac{\partial L_{EF}}{\partial \sigma} < 0 \), \( \frac{\beta - 1}{\beta} \) decreases and the opposite effect on optimal dimension follows. Moreover, totally differentiating (6) and (12) yields:

\[
\frac{\partial p_{EF}}{\partial \sigma} = \frac{\partial(\frac{\beta}{\beta - 1})}{\partial \sigma} AC + \frac{\beta}{\beta - 1} \frac{\partial AC}{\partial L_{EF}} \frac{\partial L_{EF}}{\partial \sigma} > 0 \quad \text{for } L_{EF} > \hat{L} \quad (20)
\]
\[
\frac{\partial p_{\text{NF}}}{\partial \sigma} = \frac{\partial \left( \frac{\beta}{\beta - 1} \right)}{\partial \sigma} AC + \frac{\beta}{\beta - 1} \frac{\partial AC}{\partial L_{\text{NF}}} \frac{\partial L_{\text{NF}}}{\partial \sigma} > 0 \quad \text{for } L_{\text{NF}} < \hat{L} \quad (21)
\]

By the above result and (18) it is easy to ascertain the positivity of both. In particular, if \( \sigma \to \infty \) we have that \( \beta \to 1 \) and \( \frac{\beta - 1}{\beta} \to 0 \) and no type of firm will enter in the market.

QED

6.4 Proof of Corollary 1

The slope of the entry price at \( \sigma = 0 \) can be found by evaluating (20) and (21) at \( L_{\text{EF}} = L_{\text{NF}} = \hat{L} \). Since \( AC'(\hat{L}) = 0 \) we get:

\[
\left. \frac{\partial p_{\text{EF}}}{\partial \sigma} \right|_{\sigma=0} = \left. \frac{\partial \left( \frac{\beta}{\beta - 1} \right)}{\partial \sigma} \right|_{\sigma=0} AC(\hat{L}) > 0
\]

\[
\left. \frac{\partial p_{\text{NF}}}{\partial \sigma} \right|_{\sigma=0} = \left. \frac{\partial \left( \frac{\beta}{\beta - 1} \right)}{\partial \sigma} \right|_{\sigma=0} AC(\hat{L}) > 0
\]

Then, both firms have the same slope of entry price at \( \sigma = 0 \). Differentiating (20) and (21) once more with respect to \( \sigma \) and evaluating the result at zero yields:

\[
\left. \frac{\partial^2 p_{\text{EF}}}{\partial \sigma^2} \right|_{\sigma=0} = \left. \frac{\partial^2 \left( \frac{\beta}{\beta - 1} \right)}{\partial \sigma^2} \right|_{\sigma=0} AC(\hat{L}) + \left. \frac{\beta}{\beta - 1} AC''(\hat{L}) \frac{\partial L_{\text{EF}}}{\partial \sigma} \right|_{\sigma=0}
\]

\[
\left. \frac{\partial^2 p_{\text{NF}}}{\partial \sigma^2} \right|_{\sigma=0} = \left. \frac{\partial^2 \left( \frac{\beta}{\beta - 1} \right)}{\partial \sigma^2} \right|_{\sigma=0} AC(\hat{L}) + \left. \frac{\beta}{\beta - 1} AC''(\hat{L}) \frac{\partial L_{\text{NF}}}{\partial \sigma} \right|_{\sigma=0}
\]

Since \( \frac{\partial L_{\text{EF}}}{\partial \sigma} \big|_{\sigma=0} > 0 \) and \( \frac{\partial L_{\text{NF}}}{\partial \sigma} \big|_{\sigma=0} < 0 \) we may conclude that \( \frac{\partial^2 p_{\text{EF}}}{\partial \sigma^2} \big|_{\sigma=0} > 0 \) and \( \frac{\partial^2 p_{\text{NF}}}{\partial \sigma^2} \big|_{\sigma=0} < 0 \).

QED
References


