Endogenous Switching Costs and Competition between Institutes of Higher Education

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This version: March 2005

Abstract. We consider an infinite-horizon overlapping generations model where two quality differentiated IHE’s (Institutes of Higher Education) compete by selecting the workload of their bridging program. Bridging programs are defined as extra courses the IHE’s impose on students subscribing to their master’s program after the students obtained their bachelor’s degree at the rival IHE. When IHE’s maximize enrolments but care more for their number of bachelor’s than for their number of master’s students, both the high and the low-quality IHE impose a bridging program with the same positive workload. If the IHE’s maximize total enrolments together with the average ability of their students, the high-quality IHE always imposes a bridging program while the low-quality IHE only imposes a bridging program when it cares sufficiently for the average ability of its master’s students.

Keywords: Competition, Higher Education, Endogenous switching costs

JEL Classification Numbers: I21, L15

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1 I appreciate the helpful comments of Jan Bouckaert and Wilfried Pauwels.
1 Introduction

The Bologna Declaration of June 1999 aims at establishing a two-cycle structure in Higher Education (HE) all over Europe. The common feature of this new structure is that it offers students a first final qualification (bachelor’s degree) which gives access to courses leading to a second qualification (master’s degree). So, in principle, attending a master’s program requires having obtained a bachelor’s degree. In practice, however, extra entry requirements for master’s programs vary across countries (Eurydice, 2003). In Flanders and the Netherlands, for instance, within an IHE, free access to a master’s program must be guaranteed by the completion of at least one particular bachelor’s program (Tauch and Rauhvargers, 2002). But, in these countries, an IHE is allowed to impose a bridging program on a student who switches IHE’s or who switches from professional to academic (research-oriented) education after having obtained a bachelor’s degree. In Hungary, students can switch from a college to a university conditional on taking additional subjects during the master’s program (Tauch and Rauhvargers, 2002).

In this paper we investigate whether IHE’s have an incentive to offer a bridge to students who want to switch IHE’s after having completed a bachelor’s program. We assume that the IHE’s’ objective consists of maximizing student numbers together with the quality (average ability) of the students they enroll. The well known peer group effect in (higher) education states that students perform better at school when they are among abler students (Winston, 1999). Consequently, we consider average ability as an input to the education production process and this input can only be purchased from the customers, students, in the higher education market. Moreover, students differ in terms of the quality of their input (ability); hence an IHE will not be indifferent about which student to teach and will hence try to attract high-quality students.

Some of the existing models on competition in the higher education sector (e.g. Del Rey (2002), De Fraja and Iossa (2002)) investigate the strategic behavior of public IHE’s using admission tests to control the quantity and the quality of their students. In both

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2 For Flanders, see the 4 April 2003 Flemish Parliament Act on restructuring HE in the Flemish community of Belgium. For the Netherlands, see the Netherlands Organization for International Cooperation in Higher Education or www.hbo-raad.nl. Notice that the distinction between academic and professional education seems irrelevant in some countries whereas in other countries this distinction is at the heart of the reforms.

3 Rothschild and White (1995) introduced the term customer-input technology to denote this well-known phenomenon. Notice, however, that we do not explicitly model the education production function
models the IHE’s receive an exogenous budget which they divide over teaching and research. Other models (e.g. Melnik and Shy (2003)) look at private IHE’s who use admission tests together with tuition fees in order to maximize their profits. Finally, a very recent stream of papers (e.g. Oliveira (2004) and Del Rey and Romero (2004)) deals with competition between private and public IHE’s. These IHE’s may use tests, prices or both instruments to allocate students to schools.

In this paper we focus on the competition in a HE market characterized by: absence of admission tests, uniform tuition fees determined by the government and exogenous quality differences between IHE’s. Consequently, the only instrument an IHE can handle in order to influence its number of enrolments or the quality of its enrolled students is the *workload of the bridging program* it imposes on switching students.

The paper departs from the IO literature on *system compatibility* and *endogenous switching costs*. A system can be defined as a product consisting of various parts that need to be used together but might be purchased separately (Matutes and Regibeau, 1988). Examples of systems include video game systems (which consist of a console and some system specific games), hardware and software and stereo systems. Some industries are characterized by full compatibility meaning that every part of a system produced by a firm can be used together with every part produced by another supplier (Matutes and Regibeau, 1988). Other industries produce incompatible systems so that a consumer whose preferences have changed after buying the first part of the system faces a switching cost when she wants to buy the second part from an alternative supplier. For instance, if a consumer already owns a video console of brand $i$ and wants to play a video

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4 Belgium, for instance, is one of the few countries in Europe in which IHE’s are not allowed to use admission tests to select students (except for medicine). Most of the IHE’s in other European countries admit students by their mark in the high-school leaving exam (Debande, 2003).

5 In countries like Ireland and Denmark the law prohibits the charging of tuition fees. Across the countries in which students have to pay tuition fees, there are very large differences in tuition policy. Depending on the country, tuition has to be paid up-front (Austria) or can be deferred until after graduation (Scotland). Moreover, tuition can be undifferentiated (Belgium) or differentiated on the basis of identifiable criteria. For example, in Canada tuition varies by program costs while in the US higher prestige IHE’s are allowed to charge higher tuition fees. It is however clear that there is a worldwide trend for decreased government support for higher education and hence increased costs for students in the form of some type of tuition fees (Marcucci and Johnstone, 2003).

6 In the current context we could also consider variety differentiation. This would mean that one IHE offers more professionally oriented programs while the other one offers more academically oriented programs. For more details we refer to Appendix 1.

7 Comparable to Mariñoso (2001), we assume in this paper that the different parts of a system are purchased sequentially.
game of brand $j$ she will need to buy a new video console of brand $j$ if there is total system incompatibility. If that same consumer would stay with brand $i$ she would only need to buy $i$’s game. In this example the switching cost equals the price difference between $j$’s system’s (console plus game) price and $i$’s game’s price (Mariñoso, 2001). In the switching costs literature we see an important distinction between papers on exogenous (e.g. Klemperer 1987b) and endogenous switching costs (e.g. Mariñoso (2001)). Exogenous\(^8\) switching costs have to be taken as given and are not created by the producer, while endogenous\(^9\) switching costs are artificially created by the producer for strategic purposes.

The bachelor-master structure matches the definition of a system. A student needs to obtain a bachelor’s degree before she can even consider attending a master’s program. Moreover, we suppose that students with bachelor’s degree $i$ can enter master’s program $j$ conditional on attending a bridging program installed by $j$. This gives rise to switching costs since attending the bridging program requires an extra investment of effort and time from the switching students. Moreover, these switching costs are endogenous since $j$ decides how many extra courses (workload) switching students need to attend.

Our paper offers several results. We show that increasing the workload of an IHE’s bridging program reduces the number of students who switch to that IHE for the master’s program, but that this same increase attracts more students for the IHE’s bachelor’s program. Consequently, if we assume that the IHE’s simply maximize enrolments but care more for bachelor’s than for master’s students, then we find that it is optimal for both IHE’s to subject new students to a bridging program with the same positive workload. Of course, changing the workload of a bridging program influences the quality (average ability) of the students an IHE attracts. This effect is however different for the high and the low-quality IHE. Hence, if we assume that the IHE’s maximize enrolments as well as the quality of their student body then we reach an asymmetric Nash equilibrium. Whether the high-quality IHE imposes tougher bridging programs than the low-quality one depends on how much the IHE’s care for the quality of their student body.

\(^8\) For instance, a consumer’s cost of gathering information on other suppliers of the product.

\(^9\) For instance, contract termination fees.
In the next section we state the duopoly model with endogenous switching costs for the HE sector. Afterwards, we conclude and discuss some possibilities for future research. In Appendix 1 and 2 we discuss some extensions of our model.

2 The model

2.1 The Institutes of Higher Education

Consider a model with two IHE’s, $L$ and $H$, offering exactly the same bachelor’s and master’s program except for the fact that $L$ is known for offering education of a low quality level while $H$ is known for its high-quality education. Quality is given exogenously. In Appendix 1, however, we look at the competition between a more professionally and a more academically oriented IHE offering education of the same quality level. In other words, here we assume quality differentiation while in Appendix 1 we deal with variety differentiation. In what follows, the exogenous and uniform tuition fees are normalized to zero. Consequently, IHE’s do not compete in prices. Moreover, we assume that IHE’s are not allowed to select students by admission testing. However, if a student wants to take up $i$’s ($i = L, H$) master’s program while she received her bachelor’s degree at the rival institute $j$, $i$ is allowed to impose a bridging program on this new student. Of course, an IHE’s former bachelor’s students never have to take up its bridging program; they can always enter its master’s program automatically. The only strategic variable in question in this model will thus be the workload of the bridging program of an IHE. In other words, an IHE has to decide on the degree of compatibility between its rival’s bachelor’s program and its own master’s program. IHE’s are not allowed to discriminate between the new master’s students: all students coming from the rival IHE are subjected to a bridging program with the same workload.

2.2 The students

We work within an overlapping-generations framework: each period a new generation of students of size one starts higher education and all students study for two consecutive periods. In a first period students take part in $L$ or $H$’s bachelor’s program, while in a
second period the same students enroll into L or H’s master’s program. We assume that all students succeed in both periods. Students are characterized by a parameter \( \theta \) which is assumed to be uniformly distributed in the interval \([0,1]\). When starting a bachelor’s program this parameter reflects a student’s imprecise estimate of her own innate ability level, \( \theta_b \). After the bachelor’s program, however, a student learns her true ability level, \( \theta_m \). For the rest of the paper we assume that \( \theta_b \) is independent of \( \theta_m \) for all students.

When investigating a student’s behavior we work with backwards induction. This means that we first take a look at a student’s decision process in the second period (section 2.2.1), while afterwards we discuss that same student’s decision process in the first period of her student life (section 2.2.2).

### 2.2.1 The master’s program

At the moment of choosing a master’s program all students have a certain history in higher education: either they obtained their bachelor’s degree at L or they obtained it at H. For the rest of the paper, \( n_{bL} \) represents the number of students who attend bachelor’s program L while \( n_{bH} \) denotes the number of students who complete their bachelor’s program at H. In the current section 2.2.1 we have to take the sizes of these two groups of students as given.

Remember that, in this second period of her student life, a student knows her true ability, \( \theta_m \). This true ability is independent of the estimate the student considered in the previous period, \( \theta_b \). A student knows that attending a master’s program at a higher-quality IHE leads to a gain in utility equal to \( \theta_m q_i \), but at the same time she knows that it requires a

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10 In other words, we assume that the value of attending a bachelor’s and a master’s program exceeds the cost by a large amount so that in equilibrium all students attend sequentially one bachelor’s program and one master’s program (i.e. there is market coverage).

11 Comparable to Klemperer (1987b) we could assume that for a fraction of students, \( \mu \in [0,1] \), \( \theta_s \) equals \( \theta_m \), while for the other students, \( 1-\mu \), \( \theta_s \) is independent of \( \theta_m \). This extension is investigated in Appendix 2.

12 \( n_{bH} = 1 - n_{bL} \).

13 This gain could be a monetary gain in terms of higher future earnings. In our paper, however, we do not explicitly set this gain equal to a future wage premium. First of all, because this would require discounting
higher effort cost, \((1 - \theta_m)q_i\). Depending on a student’s ability the gain outweighs the effort cost or vice versa. On top of that, a student faces an extra cost in terms of the workload of a bridging program, \(t_i\), if and only if, she switches IHE’s. In fact, in this second period of a student’s study life the student has to decide whether she prolongs her study at the same IHE or whether she switches to the other IHE. The utility function of a student who obtained her bachelor’s degree at \(i\) and who decides to continue studying at \(i\) becomes:

\[
u_i = \theta_m q_i - (1 - \theta_m)q_i
\]

This utility function has three important characteristics. First of all, an increase in quality only enhances a student’s utility when this student’s ability level lies above a certain threshold:\(^{14}\)

\[
\frac{\partial \hat{u}_i}{\partial q_i} = \theta_m - (1 - \theta_m) > 0 \Leftrightarrow \theta_m > \frac{1}{2}
\]

Secondly, we assume that a higher-ability student always gains more (or loses less) from studying at a particular IHE compared to a lower-ability student:

\[
\frac{\partial \hat{u}_i}{\partial \theta_m} = 2q_i > 0
\]

Finally, a student’s preferences satisfy a single-crossing property. This implies that the net gain from an increase in quality is higher for a higher-ability student:

\[
\frac{\partial^2 \hat{u}_i}{\partial q_i \partial \theta_m} = 2 > 0
\]

When that same student who obtained her bachelor’s degree at \(i\) decides to switch to the master’s program at \(j\), she obtains the following utility:

\[
u_j = \theta_m q_j - (1 - \theta_m)q_j - t_j
\]

since the effort cost applies to the current period while the wage premium will only be received from the next period on. Secondly, we would need to make assumptions on which part of the future wage is attributable to the quality of the IHE attended for the bachelor’s program and which to the quality of the IHE attended for the master’s program. Finally, we think it is reasonable to assume that students also take into account non-monetary gains from higher education.

\(^{14}\) The standard definition supposes that, at equal prices, variants are vertically differentiated if all consumers prefer the highest quality good (Tirole, 1988). It seems that the master’s programs in our model do not match this definition.
All the characteristics ((2), (3) and (4)) of the utility function (1) are also present in the current utility function (5). But, switching to \( j \) leads to an extra cost compared to staying with \( i \) because switching means having to attend a bridging program. Consequently, the extra cost equals the workload of the bridging program. We assume that attending a bridging program with a certain workload requires the same effort and time investment for every student independent of ability.

By comparing \( u_{Li} \) with \( u_{LH} \) we can define the ability level of a student with a bachelor’s degree obtained at \( L \) and indifferent between staying with \( L \) and switching to \( H \):

\[
\hat{\theta}_m \equiv \frac{(q_H - q_L) + t_H}{2(q_H - q_L)} \tag{6}
\]

Students with an ability level smaller than \( \hat{\theta}_m \) stay with \( L \), the other students find it worthwhile to switch to \( H \). If \( H \) sets the workload of its bridging program equal to the quality difference, no student will find it worthwhile to switch from \( L \) to \( H \):

\[
\hat{\theta}_m < 1 \iff t_H < q_H - q_L \tag{7}
\]

Comparing \( u_{H2} \) with \( u_{HH} \) yields the ability level of the student with a bachelor’s degree of \( H \) who is indifferent between staying with \( H \) and switching to \( L \):

\[
\tilde{\theta}_m \equiv \frac{(q_H - q_L) - t_L}{2(q_H - q_L)} \tag{8}
\]

We see that students with an ability level below \( \tilde{\theta}_m \) switch to \( L \) while the other students stay with \( H \). If \( L \) sets the workload of its bridging program equal to the quality difference no student switches from \( H \) to \( L \):

\[
0 < \tilde{\theta}_m \iff t_L < q_H - q_L \tag{9}
\]

Notice that without the exogenous quality difference between the two IHE’s, students have no incentive to switch and hence IHE’s have no reason to impose a bridging program.

Using (6) and (8) we can determine the number of students enrolling into master’s program \( L, n_{mL} \), and \( H, n_{mH} \). The students \( i \) attracts for its master’s program can be divided into two groups: a first group consists of former bachelor’s students who decide
to stay with i, while the second group consists of students who obtained their bachelor’s degree at j and who decide to switch to i:

\[ n_{ml} = n_{bl} \hat{\theta}_m + (1 - n_{bl}) \tilde{\theta}_m \]  
\[ n_{mlt} = (1 - n_{bl})(1 - \hat{\theta}_m) + n_{bl}(1 - \tilde{\theta}_m) \]

It follows that:

\[ \frac{\partial n_{ml}}{\partial t_L} = \frac{-1 + n_{bl}}{2(q_H - q_L)} \leq 0 \]  
\[ \frac{\partial n_{mlt}}{\partial t_H} = \frac{-n_{bl}}{2(q_H - q_L)} \leq 0 \]

PROPOSITION 1: Given the workload of j’s bridging program, an increase in the workload of i’s bridging program reduces the number of students switching to i’s master’s program, hence reduces the number of students enrolling into i’s master’s program.

Notice that, the larger the group of students with a bachelor’s degree awarded by i, the smaller the negative effects given in (12) and (13). This is due to the fact that students who obtained their bachelor’s degree at i do not have to take up a bridging program when they decide to stay with i and hence they do not care about the workload of this bridging program of i.

2.2.2 The bachelor’s program

Remember that the parameter \( \theta_b \) represents a student’s estimate about her ability before starting a bachelor’s program. Again, a student weighs the benefit, \( \theta_b q_i \), with the effort cost, \( (1 - \theta_b)q_i \), connected to attending a bachelor’s program at a particular IHE. A student acts rational when choosing a bachelor’s program. This means that she also takes into account her expected discounted next-period utility. (Students as well as IHE’s discount the future with factor \( \delta < 1 \).) We assume a student takes the current workload of a bridging program to be the best indicator for the next-period workload of the
bridging program. If a student chooses bachelor’s program $L$ she gains the following utility:

$$u_L = \theta_b q_L - (1 - \theta_b) q_L$$

$$+ \delta \left[ \int_{\theta_m}^1 \left( \theta_m q_H - (1 - \theta_m) q_H - t_H \right) d\theta_m + \delta \int_0^1 \left( \theta_m q_L - (1 - \theta_m) q_L \right) \right]$$

The first part reflects the student’s utility from attending bachelor’s program $L$ in the current period, while the part between square brackets represents her discounted expected utility in the next period where she can choose to attend master’s program $L$ or $H$. It is clear that a student’s next-period utility depends on the decision she makes in the current period. If that same student chooses bachelor’s program $H$, she obtains the following utility:

$$u_H = \theta_b q_H - (1 - \theta_b) q_H$$

$$+ \delta \left[ \int_{\theta_m}^1 \left( \theta_m q_H - (1 - \theta_m) q_H \right) d\theta_m + \delta \int_0^1 \left( \theta_m q_L - (1 - \theta_m) q_L - t_L \right) d\theta_m \right]$$

By setting $u_L$ equal to $u_H$ we can determine the number of students choosing bachelor's program $L$ and $H$:

$$n_{HL} = \frac{4(q_H - q_L)^2 + (t_H^2 - t_L^2)\delta - 2(q_H - q_L)(t_H - t_L)\delta}{8(q_H - q_L)^2}$$

$$n_{HHL} = 1 - n_{HL}$$

First of all, we see that when both IHE’s impose a bridging program with the same workload ($t_L = t_H$), both attract $\frac{1}{2}$ of bachelor’s students. Secondly, if students are not forward-looking ($\delta = 0$) both IHE’s always enroll $\frac{1}{2}$ of bachelor’s students (independent of $t_L$ and $t_H$).

Opposite to (12) and (13) we find that:

$$\frac{\partial n_{HL}}{\partial t_L} = \frac{\delta(q_H - q_L - t_L)}{4(q_H - q_L)^2} > 0$$

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15 In equilibrium these beliefs will be fulfilled.
\[
\frac{\partial(1-n_{bl})}{\partial t_H} = \frac{\delta(q_H - q_L - t_H)}{4(q_H - q_L)^2} > 0
\]

**PROPOSITION 2:** Given the workload of j’s bridging program, an increase in the workload of i’s bridging program increases the number of students choosing its bachelor’s program.

To see the intuition for this proposition let us assume that switching costs are absent \((t_L = t_H = 0)\). Then, the probability that a particular student will prefer master’s program \(H\) equals the probability that she will prefer master’s program \(L\) in the next period.\(^{16}\) Moreover, a student knows that when \(t_i\) increases she will be less likely to switch to \(i\) after the bachelor’s program. Preferences can be changed by then, hence, more students will choose bachelor’s program \(i\) when \(t_i\) increases because this allows them to avoid costly switching in the next period.

### 2.3 Graphical illustration

Figures 1, 2 and 3 graphically illustrate everything we explained so far. On the vertical axis we find a student’s estimate of her ability when choosing a bachelor’s program. On the basis of this estimate students sort themselves over \(L\) and \(H\)’s bachelor’s programs: students with a high estimate about their ability enroll at \(H\) while students with a low estimate enroll at \(L\). On the horizontal axis we find a student’s true ability level. Depending on their true ability level and on their decision made in the previous period students choose a particular master’s program. Figure 1 shows us the division of students over \(L\) and \(H\)’s bachelor’s and master’s programs when switching students do not have to attend a bridging program. Figure 2 (3) illustrates that an increase in the workload of the bridging program of \(H\) (\(L\)) leads to an increase in the number of students choosing its bachelor’s program and to a decrease in the number of students attending its master’s program.

\(^{16}\) Without switching costs \((t_L = t_H = 0)\) \(L\) and \(H\) will attract the same number of students \((n_{bl} = n_{bhl} = n_{ml} = n_{mhl} = \frac{1}{2})\).
2.4 Average ability of enrolled students

In line with the customer-input technology treated in Rothschild and White (1995), we assume that an IHE is interested in attracting high-ability students. On the one hand, notice that changing the workload of an IHE’s bridging program does not influence the true average ability of that IHE’s bachelor’s students since these students do not know their true ability. On the other hand, this change does influence the average ability of master’s students.

When looking at Figure 1, 2 and 3 we see that the average ability of the students enrolling into master’s program $i$ always consists of two components: on the one hand, the average
ability of students who received their bachelor’s degree at \(i\) and who decide to stay with \(i\), and, on the other hand, the average ability of students who received their bachelor’s degree at \(j\) and who decide to switch to \(i\). Both averages need to be weighed by the number of students in the group. Hence, the average ability of the students in the master’s programs of \(L\) and \(H\) becomes:

\[
\overline{\theta}_{mL} = \frac{n_{bl} \hat{\theta}_m \hat{\theta}_m + (1-n_{bl}) \hat{\theta}_m \hat{\theta}_m}{2n_{mL}} = \frac{n_{bl} \hat{\theta}_m^2 + (1-n_{bl}) \hat{\theta}_m^2}{2n_{mL}} \tag{19}
\]

\[
\overline{\theta}_{mH} = \frac{n_{bl} (1-\hat{\theta}_m)(1+\hat{\theta}_m) + (1-n_{bl})(1-\hat{\theta}_m)(1+\hat{\theta}_m)}{2n_{mH}} = \frac{n_{bl} (1-\hat{\theta}_m^2) + (1-n_{bl})(1-\hat{\theta}_m^2)}{2n_{mH}} \tag{20}
\]

Figures 4 and 5 illustrate how a change in the workload of an IHE’s bridging program influences the average ability of the students who enroll into that IHE’s master’s program:

![Figure 4: Effect of increase in \(t_L\) on \(\overline{\theta}_{mL}\)](image)

![Figure 5: Effect of increase in \(t_H\) on \(\overline{\theta}_{mH}\)](image)

In figure 4 we start from \(t_L = t_H = 0\) and we keep \(t_H\) fixed so that the average ability of the students staying with \(L\) remains constant. Initially, an increase in \(t_L\) decreases the average ability of students in master’s program \(L\). This stems from the fact that the highest ability students will decide to stay with \(H\) after the increase (since they are the ones who have the least to lose from staying with \(H\)). Consequently, the average ability of the group of students switching from \(H\) to \(L\) decreases. Since, at this stage, this group of switching students is still relatively large, total average ability of the students choosing master’s program \(L\) also decreases. But, from a certain point on further increases in \(t_L\) start to raise
this total average ability. This is due to the fact that an increase in $t_L$ always reduces the number of switching students. So, although the average ability of these switching students goes down, the group (the weight) becomes so small compared to the group of students staying with $L$ that total average ability goes up again.

In figure 5 we also start from $t_L = t_H = 0$ and now we keep $t_L$ fixed so that the average ability of the students staying with $L$ remains constant. At first, increasing $t_H$ raises the average ability of the students in master’s program $H$ since the lowest ability students refrain from switching. But, from a certain point on the group of switching students becomes too small and further increases in $t_H$ reduce the total average ability of the students in the master’s program of $H$.

### 2.5 The solution of the game

We assume that $L$ and $H$ act simultaneously to maximize the expected discounted value of their number of bachelor’s and master’s students and the average ability of their master’s students over an infinite horizon. Similar to the students the IHE’s use discount factor $\delta$. We specify a simple Cobb-Douglas utility function in which an IHE weighs its number of bachelor’s students, the average ability of its master’s students and its number of master’s students according to $\alpha$, $\beta$ and $1-\alpha-\beta$, respectively. The utility $L$ and $H$ derive from teaching one generation of students thus becomes:

$$U_L = \alpha \log(n_{hl}) + \delta \beta \log(\bar{\theta}_{ml}) + \delta (1-\alpha-\beta) \log(n_{ml})$$  \hspace{1cm} (21)$$

$$U_H = \alpha \log(n_{hl'}) + \delta \beta \log(\bar{\theta}_{ml'}) + \delta (1-\alpha-\beta) \log(n_{ml'})$$  \hspace{1cm} (22)$$

$$0 \leq \alpha + \beta \leq 1$$

When constructing an infinite-horizon value function we need to start at a given moment in time; hence we need an arbitrary initial market share (installed base).\textsuperscript{17} In the context of our model we would need an exogenous number of students who just finished an IHE’s bachelor’s program. To simplify our analysis, however, we assume that we start in

\textsuperscript{17} See, for instance, Borenstein, Mackie-Mason and Netz (2003).
period 0 where both IHE’s have no installed base. Consequently, the infinite-horizon value function for $L$ and $H$ becomes:

$$V_L = \left[ \alpha \log(n_{bl}) + \delta \beta \log(n_{ml}) + \delta (1 - \alpha - \beta) \log(n_{ml}) \right] \left[ \frac{1}{1 - \delta} \right]$$ (23)

$$V_H = \left[ \alpha \log(n_{bl}) + \delta \beta \log(n_{ml}) + \delta (1 - \alpha - \beta) \log(n_{ml}) \right] \left[ \frac{1}{1 - \delta} \right]$$ (24)

We look for a Nash equilibrium in which the IHE’s maximize their value function by choosing a constant workload for their bridging program in every period:

$$\max_{t_L} V_L$$

$$\max_{t_H} V_H$$ (25)

Depending on the IHE’s preferences ($\alpha, \beta, 1-\alpha-\beta$) we reach different solutions for this game between $L$ and $H$. Below we only investigate a few possible cases.

PROPOSITION 3: Let us suppose that IHE’s only care about student numbers. The unique and symmetric Nash equilibrium of the game becomes:

$$0 < t^*_L = t^*_H < \Delta$$  \(n_{bl} = n_{bl}, n_{ml} = n_{ml}\) \(\Leftrightarrow \beta = 0\) and $\alpha > \frac{1}{2}$

$$t^*_L = t^*_H = 0$$  \(n_{bl} = n_{bl}, n_{ml} = n_{ml}\) \(\Leftrightarrow \beta = 0\) and $\alpha \leq \frac{1}{2}$ (26)

Both $L$ and $H$ impose a bridging program with the same positive workload if they care more for their number of bachelor’s students than for their number of master’s students (interior solution). Otherwise, switching students are not subjected to bridging programs (corner solution). Both IHE’s always attract $\frac{1}{2}$ of bachelor’s and $\frac{1}{2}$ of master’s students.

This Proposition 3 results from Proposition 1 and 2. Remember that increasing the workload of its bridging program lowers an IHE’s number of master’s students but increases its number of bachelor’s students.

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18 In Appendix 2 we investigate the case where both IHE’s have a positive installed base.

19 We restrict our analysis to constant strategies.

20 In other words, in this case there is full compatibility.
PROPOSITION 4: Let us suppose that the IHE’s do care about the average ability of their master’s students. Moreover, we assume that they care about total enrolment but not about the proportion of bachelor’s versus master’s students. The unique but asymmetric Nash equilibrium of the game is:

\[
\begin{align*}
0 < t_L^* < \Delta & \quad (n_{bl} < n_{hl}, \ n_{mL} > n_{mH}) \quad \Leftrightarrow \beta = \text{low} \quad \text{and} \quad 2\alpha = 1 - \beta \\
0 < t_H^* < t_L^* = \Delta & \quad (n_{bl} > n_{hl}, \ n_{mL} < n_{mH}) \quad \Leftrightarrow \beta = \text{high} \quad \text{and} \quad 2\alpha = 1 - \beta
\end{align*}
\]

We find that H always imposes a bridging program with a positive workload while L only imposes one if the IHE’s care sufficiently for the average ability of their master’s students. More specifically, in that case the workload of L’s bridging program will be so high that no student finds it worthwhile to switch from H to L. The IHE with the highest workload for its bridging program always attracts the highest number of bachelor’s students but the lowest number of master’s students.

To understand this, we again need to refer to Proposition 1 and 2. On top off that, we have to remember the effect of changing the workload of an IHE’s bridging program on the average ability of the students in the master’s program illustrated with Figures 4 and 5. More specifically, if H decides to set a positive workload for its bridging program, the average ability of the students in L’s master’s program will always be highest when L makes its bridging program so tough that no student switches from H to L. This stems from the fact that when L increases the workload of its bridging program the highest ability students refrain from switching and hence stay with H.

3 Conclusion and discussion

We have developed an infinite-horizon overlapping generations model of competition between two quality differentiated IHE’s. We assume that the IHE’s cannot compete in prices and are not allowed to test a student’s ability. They are however allowed to impose a bridging program on a student who obtained her bachelor’s degree at another

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21 In fact, L creates incompatibility between the bachelor’s program of H and the master’s program of L.
IHE but who wants to take up their master’s program. In the model the IHE’s decide on the workload of their bridging program. In other words, the IHE’s must choose the degree of compatibility between their master’s program and their rival’s bachelor’s program. The Nash equilibrium of this game has been shown to depend on the IHE’s preferences.

First of all, we suppose that the IHE’s simply maximize enrolments but care more for their number of bachelor’s students than for their number of enrolments for the master’s program. Then, we find that both the high and the low-quality IHE impose a bridging program with the same positive workload on students who switch IHE’s. If the IHE’s do not care sufficiently for their number of bachelor’s students, all students can switch without being subjected to a bridging program. Hence, this case is characterized by full compatibility. Secondly, we assume the IHE’s maximize total enrolments together with the average ability of their students. Moreover, they do not care about the proportion of bachelor’s versus master’s students. In this case, we conclude that the high-quality IHE always imposes a bridging program. The workload of this program is set such that the highest ability students switch from the low to the high-quality IHE after finishing their bachelor’s program. The low-quality IHE only imposes a bridging program when it cares sufficiently for the average ability of its students. But, in that case, the workload of this program is so high that no student switches from the high to the low-quality IHE. In this last case $L$ in fact decides to make its master’s program incompatible with $H$’s bachelor’s program.

In Appendix 1 we study a similar model of competition between a more professionally oriented and a more academically oriented IHE. In Appendix 2 we give up the assumption that $\theta_m$ is independent of $\theta_b$ for all students. Moreover, we no longer assume that the IHE’s start from zero. In other words, we allow for differences in installed bases.

In the future we would, first of all, want to introduce a fraction of students who only study for one period. Can we assume that the lower ability students are the ones who leave higher education after finishing the bachelor’s program? Secondly, it will be interesting to analyze the socially optimal equilibrium. Social welfare could then be defined as the sum of the surplus of the students and the surplus of the IHE’s. Or should
we treat the IHE’s as instruments simply serving the student’s welfare (Debande en Demeulemeester, 2000)? Thirdly, it might be more realistic to make the switching cost dependent on a student’s ability level.
Appendix 1: Variety differentiation

In the paper we assume that the higher education market consists of a low and a high-quality IHE. In this appendix, however, we look at two IHE’s of the same quality level but one IHE, L, offers a professionally oriented bachelor’s and master’s program \((x = 0)\) while the other one, H, offers a more academically oriented bachelor’s and master’s program \((x = 1)\). These locations are exogenous. Students are characterized by their ideal position, \(x\), on the line between professional and academic higher education. In the first period of a student’s study life this location equals \(x_b\), but after finishing the bachelor’s program this location equals \(x_m\). For simplicity we assume that \(x_m\) is independent of \(x_b\). Notice that, a student’s effort cost rises with the distance between her own preferred location and the location of the IHE she attends.

The chronology for solving the game is similar to the one we used in the paper. Comparable to Proposition 1 and 2 we find that an increase in the workload of i’s bridging program reduces its number of enrolments for the master’s program but increases its number of bachelor’s students. Consequently, if IHE’s simply maximize student numbers the Nash equilibrium of the game between L and H equals the one we described in (27). In this appendix students are only characterized by their preference for professional versus academic higher education. This prevents us from saying anything about the average ability of an IHE’s student body. For your information we include the students’ utility functions in this case of variety differentiation.

The master’s program

\[
\begin{align*}
  u_{LL} &= w - \lambda x_m \\
  u_{HH} &= w - \lambda(1 - x_m) \\
  u_{LH} &= w - \lambda(1 - x_m) - t_H \\
  u_{HL} &= w - \lambda x_m - t_L
\end{align*}
\]

The bachelor’s program

\[
\begin{align*}
  u_L = w - \lambda x_b + \delta \left[ \int_{x_b}^{x_m} (w - \lambda x_m) dx_m + \int_{x_m}^{1} (w - (1 - \lambda x_m) - t_H) dx_m \right]
\end{align*}
\]

22 Similarities with Mix and Match…
\[ u_H = w - \lambda (1 - x_m) + \delta \left[ \int_0^1 (w - \lambda x_m - t_L)dx_m + \int_{\tilde{t}_L}^1 (w - \lambda (1 - x_m))dx_m \right] \] (33)

Appendix 2: Positive installed bases and a fraction of students with correct beliefs about their ability

Remember that \( \theta_b \) represents a student’s estimate of her ability level when choosing a bachelor’s program, while \( \theta_m \) represents that same student’s true ability level which is revealed after the bachelor’s program. Up till now we assumed that \( \theta_m \) is independent of \( \theta_b \). In this appendix, however, we assume that for a fraction of students, \( \mu \), their estimate equals their true ability level: \( \theta_b = \theta_m \). Other students’ estimate of their ability is independent of their true ability.\(^{23}\) Students do not know whether their estimate of their ability equals their true ability or not until they finish the bachelor’s program. The value of the parameter \( \mu \) is public knowledge. Consequently, at the moment of choosing a bachelor’s program a student knows that the probability that her estimate will equal her true ability equals \( \mu \).

The master’s program

When choosing a master’s program a student knows her true ability level. For a fraction of students, \( \mu \), this ability level equals the estimate of their ability they considered in the previous period and hence these students do not have a reason to switch IHE’s. In other words, these students do not make the comparison between switching and not switching. This decision process only applies to the fraction of students, \( 1-\mu \), who face a true ability different from their estimate. Consequently, the number of enrolments for each master’s program becomes:

\[ n_{mL} = \mu n_{bl} + (1 - \mu)(n_{bl, \hat{\theta}_m} + (1 - n_{bl, \hat{\theta}_m})) \] (34)

\[ n_{mH} = \mu (1 - n_{bl}) + (1 - \mu)(n_{bl, (1 - \hat{\theta}_m)} + (1 - n_{bl})(1 - \hat{\theta}_m)) \] (35)

Notice that an increase in \( t_i \) still lowers the number of enrolments for \( i \)’s master’s program. But, this effect becomes smaller for larger values of \( \mu \), since students with a

\(^{23}\) Comparable to Klemperer (1987b).
correct belief about their ability do not consider switching and hence can no longer be influenced by changing the workload of a bridging program.

The bachelor’s program
At the moment of choosing a bachelor’s program a student knows that the probability that her estimate about her own ability will appear to be correct in the next period equals $\mu$. Consequently, $u_L$ and $u_H$ become:

\[
\begin{align*}
    u_L &= \theta b q_L - (1 - \theta b) q_L \\
    &= \theta b (1 - \theta b) q_L \\
    &= \theta b (1 - \theta b) q_L + (1 - \mu) \left( \int_0^1 (\theta m q_H - (1 - \theta m) q_H - t_H ) d\theta + \int_0^1 (\theta m q_L - (1 - \theta m) q_L ) d\theta \right) \\
    u_H &= \theta H q_H - (1 - \theta H) q_H \\
    &= \theta H (1 - \theta H) q_H + (1 - \mu) \left( \int_0^1 (\theta m q_H - (1 - \theta m) q_H ) d\theta + \int_0^1 (\theta m q_L - (1 - \theta m) q_L - t_L ) d\theta \right)
\end{align*}
\]

Comparing $u_L$ and $u_H$ leads to the following expression for the number of students choosing bachelor’s program $L$:

\[
n_{bL} = \left[ \frac{4(q_H - q_L)^2 (1 + \delta \mu) - (t_H^2 - t_L^2) \delta (-1 + \mu) + 2(q_H - q_L)(t_H - t_L) \delta (-1 + \mu)}{8(q_H - q_L)^2 (1 + \delta \mu)} \right]
\]

An increase in $t_i$ still raises the number of enrolments for i’s bachelor’s program but this effect becomes smaller for larger values of $\mu$. The larger the probability a student has constant beliefs, the less she cares for the workload of a bridging program when choosing a bachelor’s program.

Solution of the game
To simplify the analysis we assume that the IHE’s do not care about the average ability of their student body ($\beta = 0$). Hence, their utility function only depends on their number of bachelor’s and master’s students. We no longer assume that the IHE’s start from zero. Instead, we assume that at the moment the infinite-horizon value function starts, $L$ awarded a bachelor’s degree in the previous period to a group of students with an exogenous size of $d_0$. $H$ recently awarded a bachelor’s degree to a group of students with size $(1-d_0)^{24}$. The IHE’s maximize:

\[\text{Comparable to, for instance, Borenstein, Mackie-Mason and Netz (2003).}\]
\[
\text{Max}_{t_L} V_L = (1 - \alpha) \log[\mu d_0 + (1 - \mu) (d_0 \hat{\theta}_m + (1 - d_0) \tilde{\theta}_m)] + \\
\left[\alpha \log[n_{hl}] + \delta (1 - \alpha) \log[n_{ml}] \right] \left[ \frac{1}{1 - \delta} \right]
\]

\[
\text{Max}_{t_H} V_H = (1 - \alpha) \log[\mu (1 - d_0) + (1 - \mu) (d_0 (1 - \hat{\theta}_m) + (1 - d_0) (1 - \tilde{\theta}_m))] + \\
\left[\alpha \log[n_{hl}] + \delta (1 - \alpha) \log[n_{ml}] \right] \left[ \frac{1}{1 - \delta} \right]
\]

The Nash equilibrium of this game becomes:

\[0 < t^*_L < t^*_H < \Delta \quad \iff \quad d_0 < \frac{1}{2} \quad \text{and} \quad \alpha > \frac{1}{2}\]

\[0 < t^*_H < t^*_L < \Delta \quad \iff \quad d_0 > \frac{1}{2} \quad \text{and} \quad \alpha > \frac{1}{2}\]

\[t^*_L = t^*_H = 0 \quad \iff \quad \alpha \leq \frac{1}{2}\]

Suppose IHE’s care sufficiently for bachelor’s students (\(\alpha > \frac{1}{2}\)). Hence, we first of all conclude that differences in installed bases lead to asymmetric equilibria in the workloads of bridging programs. The IHE with the largest installed base sets a higher workload for its bridging program compared to its competitor since the students in its installed base do not care about the workload of its bridging program. Moreover, this implies that the IHE with the largest installed base enrolls more bachelor’s but less master’s students than its competitor. Secondly, notice that an increase in the fraction of students with correct beliefs about their ability induces IHE’s to set a lower workload for their bridging programs. This stems from the fact that students with correct beliefs do not consider switching after the bachelor’s program and can hence (at that moment) not be influenced by changing the workload of a bridging program. Again, if the IHE’s attach a lot of importance to their number of enrolments for the master’s program (\(\alpha \leq \frac{1}{2}\)), no student will be subjected to a bridging program (full compatibility).
References


