On Choice of Technique and Development Planning: Optimal and Stiglitz Policies in the RSS Model*

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September 2003; August 2008

Abstract: In earlier work, the authors have presented a complete characterization of Stiglitz policies for the RSS model in an undiscounted setting with linear felicity functions: Such policies are optimal when a suitably defined sufficient statistic $\xi_\sigma \leq 1$, uniquely optimal when $\xi_\sigma < 1$, and bad when $\xi_\sigma > 1$. It has been an open question as to what are the optimal policies when Stiglitz policies are bad, and in this paper, we give a complete characterization of optimal programs when $\xi_\sigma > 1$. Our solution draws on the recently developed theory of undiscounted dynamic programming for the RSS model.

Journal of Economic Literature Classification Numbers: D90, C62, O21.

Key Words: Choice of technique, development planning, overtaking criterion, optimal program, golden-rule stock, golden-rule price, value-loss decomposition, undiscounted dynamic programming.

Running Title: Optimal Policy Correspondence

This is an abstract and preliminary notes that form a background for a rather general talk by M. Ali Khan on 'economic development' at the the Faculty of Economics, *University of Porto* Portugal sometime in April, 2009.

*The original version was initiated when Khan visited the Department of Economics at Cornell during July 7-14, 2003; the authors are grateful to the *Center for Analytic Economics* at Cornell and to the *Center for a Livable Future* at Johns Hopkins for research support. This version was prepared when Khan was visiting the Department of Economics, National University of Singapore, July-December 2008.

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1 Introduction

Robert Solow's (2000) call for an analysis of Ramsey optimal growth under the technological specification of a model due to Robinson, Solow and Srinivasan, henceforth RSS model, has been by now almost fully answered. Under the identification of "Ramseyian bliss" as the utility level attained by the golden-rule stock, and under the assumption of this level being attained by a single type of machine, say σ, it has been shown that an optimal program starting from any arbitrary initial stock exists and converges to an identifiable subset of the transition set, the McKenzie facet. If the felicity function is strictly concave, as opposed to being linear, this facet shrinks to a point, and we obtain convergence to the bliss utility levels, and to the consumption and investment levels that go with them. These results are direct applications of the general theory of intertemporal resource allocation as developed by Gale (1967), Brock (1970) and McKenzie (1986, 2002).[1]

Indeed, given the specific structure of the RSS model, it is possible to supplement this application of the general theory by results on transition dynamics. A specific decision rule, the so-called Stiglitz policy presented in Stiglitz (1968), can be identified, and optimality requires the Ramsey planner to determine on the basis of the golden-rule type of machine σ, a set of desirable types of machines, use them for consumption if they are available, and allocate any remaining labor to the production of the σ. What is of considerable interest is that model delineates circumstances under which the Stiglitz policy, however intuitively natural it may seem as a rough rule-of-thumb, is far from being optimal. Even in the setting of a single type of machine, and with a linear felicity function, a simple example shows that the Stiglitz policy can be bad. Furthermore, with a piecewise linear felicity function, and two types of machines, an example can be constructed in which optimality requires the planner to produce the non-golden-rule type of machine, the type other than σ.[2] However, the first example can be overcome by identifying another policy that is optimal, one in which the planner, for a determinable non-negligible set of stocks, builds more machines even when he has a surplus so that the long-run level may be attained as fast as possible. Thus, at least for an economy in which the 'choice of technique' is not at issue, the so-called two-sector RSS model, the characterization of optimality with a linear felicity function, is complete.

However, the question as to the transition dynamics remains totally open when the 'choice of technique' is at issue. We do not know how the Ramsey planner is to proceed in all those situations when the Stiglitz policy is bad, or to phrase the problem in a more concrete vernacular, we do not know what are optimal choices of technique over time when the marginal rate of transformation corresponding to the golden-rule type of machine σ, ξ_σ, is greater than one. This has proved to be a particularly recalcitrant problem for the full RSS model even for the case of linear felicities. Thus, Solow's (2000) call remains to be fully answered; or to go back almost a half-century, the research program laid out by Solow-Samuelson (1956) in the conclusion to their multi-sectoral Ramsey analysis remains to be worked

[1] For details as to these results, as well as a genealogy of the RSS model, the reader is referred to Khan-Mitra (2005). However, the references given there need to be supplemented by Robinson (1956), Little (1957) and Morishima (1969).

[2] See Examples 2 and 3 in Khan-Mitra(2005).

out.

> Various specializations of the foregoing analysis are also possible as e.g., the important case where the production function f is homogeneous of the first degree. The Leontief-Koopmans-von Neumann case of dynamic *linear* finite activities analysis can also be considered, only here we must replace the Euler equalities by more general differential *inequations.* Finally, replacing continuous time by discrete time, integrals by sums, and derivatives by differences would bring us back to the discrete case from which Euler deduced his extremal condition as a limit, but no one seems to have worked out the full Hamiltonian theory for this discrete case.

This research program involves both the discounted and the undiscounted theory with both linear and strictly concave felicity functions give full attention, and while it is in the process of completion, this paper limits itself to a complete answer to the question that has been posed above: the optimal policy for the undiscounted case with linear felicities and $\xi_\sigma > 1$. A precise and detailed discussion of this solution will be offered in the sequel; here we simply point out that it builds on the solution of the two-sector RSS model as well as on that for the (full) RSS model. From the first, we use the pan-map as our relevant point of departure, and from the second, the fact that the desirability of machine types is independent of initial temporal conditions. Whereas geometry remains an important illustrative device, it no longer serves as an engine of analysis, as in the earlier work Khan-Mitra (2007); and whereas value-loss considerations are always in the background, unlike Khan-Mitra (2005), period-by-period value-loss considerations are no longer relevant, and we have to draw on the theory of undiscounted dynamic programming developed in Khan-Mitra (2006). Even though this theory is developed for the two-sector RSS case, it is tailored for a situation when all good programs satisfy the turnpike property, rather than the average turnpike property. Since this is the precisely the situation when $\xi_\sigma > 1$, it finds a ready extension and application to the problem at hand. As is well-understood by readers familiar with dynamic programming, the trick is to guess the optimal policy and then use the Bellman-Blackwell functional equation to verify that it is indeed optimal. And so the novelty of the result presemted below, and the principal contribution of this paper, lies in characterizing the optimal policy when the Stiglitz policies represent a total failure. How the precise, and rather intricate, policy prescriptions mesh with the Keynes-Ramsey intuition for the one-sector model, remains an open question.

What we can say is that the solution that we present, in drawing on the important distinction between choice of technique that is appropriate in the short-run from that which is appropriate in the long-run, connects to the literature of the sixties on planning in India (and elsewhere). This literature[3] comes as close to stating the problem as precisely as can be expected in the pre-Pontryagin period.

> [T]he whole point of the exercise is to get alternative time-paths of consumption, out of which choice can be made according to circumstances. ... So, the choice, in this system,

[3] In addition to Raj-Sen (1961) and Sen (1960) in particular, also see Dobb (1956, 1960, 1961, 1967), Halevi (1987), Mirrlees (1962), Naqvi (1963), Solow (1962a); and for an open-economy perspective, Bardhan (1971).

should not be put so much as one between a faster and a slower growth rate, but as a choice between constant consumption, or its constant rate of growth, or its constant rate of growth of rate of growth, and so on. It should be noted that from the long-run point of view, each case is better than the earlier one, though the position is the reverse in the short period. Which type of policy we choose will depend very much on the time necessary for the long-run effects to compensate the short-run results.[4]

We conclude this introduction with a schematic outline of the paper. In section 2 we present the model and the antecedent resulst that we'll be invoking. Section 3 recalls the principal result from the theory of undiscounted dynamic programming theory. Section 4 is a rather full description of the optimal policy and a presentation of the principal result. The concluding Section 5 identifies problems that remain open. The technical and computational details of the proofs are collected in an Appendix.

2 The Model and Antecedent Results

We begin with some preliminary notation. Let $\mathbb{N}$ ($\mathbb{N}_+$) be the set of non-negative (positive) integers, $\mathbb{R}$ ($\mathbb{R}_+$) the set of real (non-negative) numbers. We shall work in finite-dimensional Euclidean space $\mathbb{R}^n$ with non-negative orthant $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n : x_i \geq 0,\ i = 1, \cdots, n\}$. For any x, y in $\mathbb{R}^n$, let the inner product $xy = \sum_{i=}^{n} x_i y_i$, and $x >> y,\ x > y,\ x \geq y$ have their usual meaning. Let $e(i),\ i = 1, \cdots, n$, be the i^{th} unit vector in $\mathbb{R}^n$, and e be an element of $\mathbb{R}^n_+$ all of whose coordinates are unity. For any $x \in \mathbb{R}^n$, let $\|x\|$ denote the Euclidean norm of x. The empty set is denoted by $\emptyset$ and set-theoretic subtraction between A and B by A/B.

Our choice of $\mathbb{R}^n$ is dictated by the consideration of an economy capable of producing a finite number n of alternative types of machines. For every $i = 1, \cdots, n$, one unit of machine of type i requires $a_i > 0$ units of labor to construct it, and together with one unit of labor, each unit of it can produce $b_i > 0$ units of a single consumption good. Thus, the production possibilities of the economy can be represented by an (labor) input-coefficients vector, $a = (a_1, \cdots, a_n) >> 0$ and an output-coefficients vector, $b = (b_1, ..., b_n) >> 0$. Without loss of generality[5] we shall assume that the types of machines are numbered such that $b_1 \geq b_2 \cdots \geq b_n$.

We shall assume that all machines depreciate at a rate $d \in (0, 1)$. Thus the effective labor cost of producing a unit of output on a machine of type i is given by $(1 + da_i)/b_i$: the direct labor cost of producing unit output, and the indirect cost of replacing the depreciation of the machine in this production.[6] We shall work with the reciprocal of the effective labor cost, the effective output that takes the depreciation into account, and denote[7] it by c_i for the machine of type i. Throughout this

[4]See Raj-Sen (1961, pp. 48, 51 and concluding section). We remind the reader that an earlier version of the Raj-Sen paper (with a different title) was published in *Arthaniti* in 1959, and that Naqvi (1963) is a follow-up to the Raj-Sen paper as his leading footnote clearly indicates.

[5]Note that Stiglitz (1968) assumes that $b_i > b_j$ implies that $a_i > a_j$; whereas this is a natural hypothesis, we make no such assumption.

[6]See Stiglitz (1968, pp. 608-609) on a "labor theory of value" interpretation.

[7]As we shall see below, c_i is the value of the steady-state consumption per man if only machines of type i are used and produced, a consideration that governs our choice of notation.

paper, we shall assume that there is a unique machine type σ at which this effective labor cost $(1+da_i)/b_i$ is minimized, or at which the effective output per man $b_i/(1+da_i)$ is maximized. Thus, we shall assume:

$$\text{There exists } \sigma \in \{1, \cdots, n\} \text{ such that for all } i = 1, \cdots, n,\ i \neq \sigma,\ c_\sigma > c_i. \tag{1}$$

For each date $t \in \mathbb{N}$, let $x(t) = (x_1(t), \cdots, x_n(t)) \geq 0$ denote the amounts of the n types of machines that are available in time-period t, and let $z(t+1) = (z_1(t+1), \cdots, z_n(t+1)) \geq 0$ be the *gross investments* in the n types of machines during period $(t+1)$. Hence, $z(t+1) = (x(t+1) - x(t)) + dx(t)$, the sum of net investment and of depreciation. Let $y(t) = (y_1(t), ..., y_n(t))$ be the amounts of the n types of machines used for production of the consumption good, $by(t)$, during period $(t+1)$.[8] Let the total labor force of the economy be stationary and positive. We shall normalize it to be unity. Clearly, gross investment, $z(t+1)$ representing the production of new machines of the various types, will require $az(t+1)$ units of labor in period t. Also, $y(t)$ representing the use of available machines for manufacture of the consumption good, will require $ey(t)$ units of labor in period t. Thus, the availability of labor constrains employment in the consumption and investment sectors by $az(t+1) + ey(t) \leq 1$. Note that both the flow of consumption and of investment (new machines) are in gestation during the period and available at the end of it. We now give a formal summary of this technological structure.

Definition 1 *A program from x_o in $\mathbb{R}^n_+$ is a sequence*[9] $\{x(t), y(t)\}$ *with* $(x(t), y(t)) \in \mathbb{R}^n_+ \times \mathbb{R}^n_+$ *such that* $x(0) = x_o$, *and for all* $t \in \mathbb{N}$,

$$x(t+1) \geq (1-d)x(t),\ 0 \leq y(t) \leq x(t),\ a(x(t+1) - (1-d)x(t)) + ey(t) \leq 1.$$

A program $\{x(t), y(t)\}$ *is simply a program from* $x(0)$.

Definition 2 *Associated with any program* $\{x(t), y(t)\}$ *is a gross investment sequence* $\{z(t+1)\}$ *with* $z(t+1) \in \mathbb{R}^n_+$, *and a consumption sequence* $\{by(t)\}$ *such that for all* $t \in \mathbb{N}$,

$$z(t+1) = x(t+1) - (1-d)x(t).$$

Definition 3 *A program* $\{x(t), y(t)\}$ *is called stationary if for all* $t \in \mathbb{N}, (x(t), y(t)) = (x(t+1), y(t+1))$.

The preferences of the planner are represented by a felicity function, $w : \mathbb{R}_+ \longrightarrow \mathbb{R}$, which is assumed to be continuous, strictly increasing and concave, and differentiable.[10] We suppose, as in the literature taking its lead from Ramsey (1928), that future welfare levels are treated like current ones in the planner's objective function. The notion of optimality that we use is due to Brock (1970), and the notion of "overtaking" is due to Atsumi (1965) and von Weiszäcker (1965).[11]

[8] The reader may choose to think of the consumption in period t as the scalar $c(t+1)$, with c_i reserved for $b_i/(1+da_i)$, we avoid this notation in the text to prevent any ambiguity.

[9] Note $\{x(t), y(t)\}$ is an abbreviation of $\{x(t), y(t)\}_{t\in\mathbb{N}}$; we use it for notational simplicity.

[10] We leave it to the reader to check that differentiability of w is not needed, and derivatives of w can be replaced uniformly by (for example) the right hand derivative of w. These exist since w is concave and the point of evaluation of the (right-hand) derivative is always positive.

[11] Brock (1970) uses the terminology of "weakly maximal" programs for what we call optimal programs. The notion of optimality in Atsumi and von Weiszäcker is stronger, and creates problems in proving existence of optimal programs in many reasonable models.

Definition 4 *A program $\{x^*(t), y^*(t)\}$ from x_o is called optimal if*

$$\liminf_{T\to\infty} \sum_{t=1}^{T} [w(by(t)) - w(by^*(t))] \leq 0$$

for every program $\{x(t), y(t)\}$ from x_o. It is called a stationary optimal program if it is stationary and optimal.

Note that the optimality notion can be restated to say that there does not exist any other program $\{x(t), y(t)\},\ x(0) = x_o$, a number $\varepsilon > 0$ and a time period t_ε such that $\sum_{t=1}^{T}[w(by(t)) - w(by^*(t))] > \varepsilon$ for all $T \geq t_\varepsilon$. Thus an optimal program is one in comparison to which no other program from the same initial stock is eventually significantly better, for any given level of significance.

Define the *transition possibility set* Ω as a collection of pairs (x, x'), such that it is possible to obtain the amounts of the n types of machines x' in the next period (tomorrow) from the amounts of the n types of machines x available in the current period (today). Formally,

$$\Omega = \{(x, x') \in \mathbb{R}^n_+ \times \mathbb{R}^n_+ : x' - (1-d)x \geq 0 \text{ and } a(x' - (1-d)x) \leq 1\}.$$

For any $(x, x') \in \Omega$, one can consider the amounts y of the n types of machines available for the production of the consumption good. Formally, we have a correspondence $\Lambda : \Omega \longrightarrow \mathbb{R}^n_+$ given by

$$\Lambda(x, x') = \{y \in \mathbb{R}^n_+ : 0 \leq y \leq x \text{ and } ey \leq 1 - a(x' - (1-d)x)\}.$$

For any $(x, x') \in \Omega$, we shall denote the number of machines that are produced in the period $(x' - (1-d)x)$ by z. Note that $z \geq 0$. Finally, the *reduced form utility function,* $u : \Omega \longrightarrow \mathbb{R}_+$, is defined on Ω such that

$$u(x, x') = \max\{w(by) : y \in \Lambda(x, x')\}.$$

We leave it to the reader to check for herself that our assumptions on w imply that the reduced form utility function, u, is upper semicontinuous and concave on Ω, and that it is increasing in its first argument and decreasing in its second argument.

Given the description of the transition possibility set Ω, and of the reduced form utility function, u, it is clear that for any program $\{x(t), y(t)\}$ from x_o, $(x(t), x(t+1)) \in \Omega$ and $y(t) \in \Lambda(x(t), x(t+1))$ for all $t \in \mathbb{N}$. Also, for any optimal program $\{x^*(t), y^*(t)\}$ from x_o, $w(by^*(t)) = u(x^*(t), x^*(t+1))$ for all $t \in \mathbb{N}$, and for every program $\{x(t), y(t)\}$ from x_o,

$$\liminf_{T\to\infty} \sum_{t=0}^{T} [u(x(t), x(t+1)) - u(x^*(t), x^*(t+1))] \leq 0.$$

In summary, the basic data of the model denoted by the triple $(w, (a_i, b_i)_{i=1}^n, d)$ summarizing the felicity function w, the technology $(a_i, b_i)_{i=1}^n$, and the depreciation rate d, is converted to the pair (u, Ω) summarizing the reduced-form utility function u and the transition possibility set Ω.

We begin with a definition.

Definition 5 A golden-rule stock is $\hat{x} \in \mathbb{R}^n_+$ such that $(\hat{x}, \hat{x})$ is a solution to the problem: maximize $u(x, x')$ subject to (i) $x' \geq x$, (ii) $(x, x') \in \Omega$.

If we limit ourselves to a stationary program in which only a machine of type i is used and produced, the constraint of labor allows us to maintain the stock[12] $(1/(1 + da_i)$ and obtain a stationary consumption stream in the amount $b_i/(1 + da_i) = c_i$. Since we have assumed (in (1) above) that a machine of type σ is the one that uniquely minimizes effective labor costs, we see that it is also the type that uniquely maximizes the consumption per unit of labor.[13] Denote $\hat{y} = (1/(1 + da_\sigma))e(\sigma)$, and note that if we are in such a stationary state, $b\hat{y} = (b_\sigma/(1 + da_\sigma))$ and $w'(b\hat{y})$ the marginal utility of output produced. Furthermore, since the labor cost of a machine of type i is a_i, and a unit of labor is worth $((1 + da_i)/b_i)^{-1}$ units of output, a machine is worth $a_i \times (b_i/(1 + da_i)$ in terms of output, and $w'(b\hat{y})(a_i \times (b_i/(1+da_i)))$ in terms of utils. We can then identify a stationary price system ($\hat{q}$ in terms of the consumption good and $\hat{p}$ in terms of utils)[14] for the various types of machines as $\hat{q}_i = (a_i b_i/(1+da_i))$ and $\hat{p}_i = w'(b\hat{y})\hat{q}_i$ for each $i = 1, \cdots, n$.

We can now turn to a portmanteau theorem.

Theorem 1 *(i) There exists a unique golden-rule stock* $\hat{x} = (1/(1+da_\sigma))e(\sigma)$. *(ii) For any arbitrary initial stock,* $x_o \in \mathbb{R}^n_+$, *there exists an optimal program from* x_o. *If the initial stock* x_o *equals* $\hat{x} = \hat{y} = (1/(1+da_\sigma))e(\sigma)$, *then the stationary program* $\{\hat{x}, \hat{y}\}$ *is an optimal program from* x_o. *(iii) Any optimal program* $\{x(t), y(t)\}$ *converges to the von Neumann facet, and thus* $\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} y_i(t) = \lim_{t\to\infty} z_i(t) = 0$ *for all* $i \neq \sigma$. *If the felicity function* $w(\cdot)$ *is strictly concave,* $\lim_{t\to\infty} x(t) = \lim_{t\to\infty} y(t) = (1/1 + da_\sigma)e(\sigma)$ *and* $\lim_{t\to\infty} z(t) = (da_\sigma/1 + da_\sigma)e(\sigma)$.

Definition 6 *A program* $\{x(t), y(t)\}$ *is called good if there exists* $G \in \mathbb{R}$ *such that* $\sum_{t=0}^{T}(w(by(t)) - w(b\hat{y})) \geq G$ *for all* $T \in \mathbb{N}$. *A program is called bad if* $\lim_{T\to\infty} \sum_{t=0}^{T}(w(by(t)) - w(b\hat{y})) = -\infty$.

For any $y \in \Lambda(x, x')$ and any $(x, x') \in \Omega$, let

$$\delta_{(\hat{x},\hat{p})}(x, x') = w(b\hat{y}) - w(by) - \hat{p}(x' - x) = \hat{p}(x - x') - (w(by) - w(b\hat{y})). \tag{2}$$

Whenever there is no possibility of confusion, we shall abbreviate $\delta_{(\hat{x},\hat{p})}(x(t), x(t + 1))$ by $\delta(t)$ for any program $\{x(t), y(t)\}$. We shall refer to $\{\delta(t)\}$ as the value-loss sequence associated with the program $\{x(t), y(t)\}$.

Proposition 1 *The value-loss sequence* $\{\delta(t)\}_{t\in\mathbb{N}}$ *of any program* $\{x(t), y(t)\}$ *is non-negative, and*

$$\sum_{t=0}^{T}(w(by(t)) - w(b\hat{y})) = \hat{p}(x(0) - x(T + 1)) - \sum_{t=0}^{T}\delta(t) \text{ for all } T \in \mathbb{N}.$$

[12] The labor requirements of the consumption sector in the amount $(1/(1 + da_i)$ plus those of the investment sector arising from replacement for depreciation in the amount $da_i/(1 + da_i)$ add up to the total labor available.

[13] As alluded to in Footnote 7 above.

[14] When the felicity function is linear, the magnitudes of $\hat{p}$ and $\hat{q}$ are identical, though their units remain different. Note also the identities $q_i = a_i c_i$ and $c_i + dq_i = b_i$ for all i.

Proposition 2 *(i) Any program that is not good is bad. (ii) For any initial stock* $x_0 \in R^n_+$ *there exists good program* $\{x(t), y(t)\}_{t=0}^{\infty}$ *such that* $x(0) = x_0$. *(iii) A program* $\{x(t), y(t)\}_{t=0}^{\infty}$ *is good if and only if* $\sum_{t=0}^{\infty} \delta(x(t), y(t), x(t+1)) < \infty$.

We now define the aggregate value-loss associated with any program as

$$\Delta(x_o) = \inf\{\sum_{t=0}^{\infty} \delta(t) : \{x(t), y(t)\} \text{ is a program from } x_o\}.$$

Our next two results assert that this infimum is a finite number and that it can be attained.

Choice of Techniques in the Long-Run We are now in a position to describe what the economy looks like in the long-run. Towards this end, we begin with a characterization of the von Neumann facet as described in McKenzie (1968, 1986). It is of interest that under our standing hypothesis as described in (1), this reduces to a line in the Euclidean space of dimension $2n$. It is of interest that under our standing hypothesis as described in (1), this reduces to a line in the Euclidean space of dimension $2n$.

Lemma 1 *The von Neumann facet* $\{(x, x') \in \Omega : \delta_{(\hat{p},\hat{x})}(x, x') = 0\}$ *is a subset of* $\{(x, x') \in \Omega : x'_i = x_i = 0, i \neq \sigma,\ x'_\sigma = (1/a_\sigma) + \xi_\sigma x_\sigma\},\ \xi_\sigma = 1 - d - (1/a_\sigma)$, *with equality if the felicity function* w *is linear. If the felicity function is strictly concave, the facet is the singleton* $\{(\hat{x}, \hat{x})\}$.

Theorem 2 *Assume that for each good program* $\{u(t), v(t)\}_{t=0}^{\infty}$

$$\lim_{t \to \infty} (u(t), v(t)) = (\widehat{x}, \widehat{x}).$$

Then for each program $\{x(t), y(t)\}_{t=0}^{\infty}$ *the following conditions are equivalent: (i)* $\sum_{t=0}^{\infty} \delta(x(t), y(t), x(t+1)) = \Delta(x(0))$. *(ii)* $\{x(t), y(t)\}_{t=0}^{\infty}$ *is optimal. (iii)* $\{x(t), y(t)\}_{t=0}^{\infty}$ *is maximal.*

References

Atsumi, H. (1965): "Neoclassical Growth and the Efficient Program of Capital Accumulation." *Review of Economic Studies* 32: 127-136.

Brock, W. A. (1970). "On Existence of Weakly Maximal Programmes in a Multi-Sector Economy." *Review of Economic Studies* 37: 275-280.

Dobb, M. (1956): "Second Thoughts on Capital-intensity of Investment." *Review of Economic Studies* 24: 33-42.

Dobb, M. (1960): *An Essay on Economic Growth and Planning.* New York: Monthly Review Press.

Dobb, M. (1961): Review of A. K. Sen's *Choice of Techniques, Kyklos* 14: 1961.

Dobb, M. (1967).: "The Question of 'Investment-priority for Heavy Industry'." In *Capitalism, Development and Planning,* Chapter IV. New York: International Publishers.

Gerschenkron, A. (1962): *Economic Backwardness in Historical Perspective, a Book of Essays.* Cambridge: Belknap Press.

Gerschenkron, A. (1970): *Europe in the Russian Mirror: Four Lectures in Economic History.* Cambridge: Cambridge University Press.

Gerschenkron, A. (1977): *An Economic Spurt that Failed: Four Lectures in Austrian History.* Princeton: Princeton University Press.

Halevi, J. (1987): "Investment planning." In *The New Palgrave,* edited by J. Eatwell, P. K. Newman and M. Milgate. London: MacMillan.

Khan, M. Ali (2000). "Srinivasan on Choice of Technique again." Chapter 2 in Ranis-Raut (2000).

Khan, M. Ali and Mitra, T. (2005): "On Choice of Technique in the Robinson-Solow-Srinivasan model." *International Journal of Economic Theory* (2005) 1:83-109.

Khan, M. Ali and T. Mitra (2006). "Undiscounted Optimal Growth under Irreversible Investment: A Synthesis of the Value-loss Approach and Dynamic Programing." *Economic Theory* 29:341-362.

Khan, M. Ali and T. Mitra (2007a). "Optimal Growth in a Two-Sector Model without Discounting: A Geometric Investigation." *Japanese Economic Review* 58:191-225.

Khan, M. Ali and T. Mitra (2007b). "Optimal Growth under Discounting in the Two-sector Robinson-Solow-Srinivasan Model: A Dynamic Programming Approach." *Journal of Difference Equations and Applications* 13:151-168.

Khan, M. Ali and T. Mitra (2008). "Growth in the Robinson-Solow-Srinivasan Model: Undiscounted Optimal Policy with a Strictly Concave Welfare Function." *Journal of Mathematical Economics* 44: 707-732.

McKenzie, L. W. (1968): "Accumulation Programs of Maximum Utility and the von Neumann Facet." In *Value, Capital and Growth,* edited by J. N. Wolfe, pp. 353-383. Edinburgh: Edinburgh University Press.

McKenzie, L. W. (1986) "Optimal Economic Growth, Turnpike Theorems and Comparative Dynamics." In *Handbook of Mathematical Economics,* Volume 3, edited by K. J. Arrow and M. Intrilligator, pp. 1281-1355. New York: North-Holland Publishing Company.

McKenzie, L. W. (2002): *Classical General Equilibrium Theory.* Cambridge: MIT Press,

Metcalf, C. (2004): "On the Perils of a Stiglitz Policy in the RSS model." Mimeo, The Johns Hopkins University.

Mirrlees, J. A. (1962): "Choice of Techniques." *Indian Economic Review* 6: 93-102.

Morishima, M. (1969): *The Theory of Growth.* Oxford: Oxford University Press.

Naqvi, K. A. (1963): "Machine Tools and Machines: a Physical Interpretation of Marginal Rate of Saving." *Indian Economic Review* 7: 19-28.

Nishimura, K., Yano, M. (1995a): "Durable Capital and Chaos in Competitive Business Cycles." *Journal of Economic Behavior and Organization* 27: 165-181.
Okishio, N. (1966): "Technical Choice under Full Employment in a Socialist Economy." *Economic Journal* 76: 585-592.
Okishio, N. (1987): "Choice of Technique and the Rate of Profit." In *The New Palgrave,* edited by J. Eatwell, P. K. Newman and M. Milgate. London: MacMillan.
Raj, K. N. and Sen, A. K. (1961). 'Alternative Patterns of Growth under Conditions of Stagnant Export Earnings', *Oxford Economic Papers,* vol. 13, pp. 43-52.
Ramsey, F. (1928).: "A Mathematical Theory of Savings." *Economic Journal* 38: 543-559.
Robinson, J. (1956): *The Accumulation of Capital.* London: MacMillan.
Robinson, J. (1960): *Exercises in Economic Analysis.* London: MacMillan.
Robinson, J. (1969): "A Model for Accumulation proposed by J. E. Stiglitz." *Economic Journal* 79: 412-413.
Sen, A. K. (1960).: *Choice of Techniques.* Oxford: Basil Blackwell.
Sen, A. K. (1968): "Optimal Savings, Technical Choice and the Shadow Price of Labour." Introduction to the third edition of Sen (1960).
Solow, R. M. (1962a): "Some Problems in the Theory and Practice of Economic Planning." *Economic Development and Cultural Change* 10: 216-222.
Solow, R. M. (2000): "Srinivasan on Choice of Technique." Chapter 1 in Ranis-Raut (2000).
Srinivasan, T. N. (1962a): "Investment Criteria and Choice of Techniques of Production." *Yale Economic Essays* 1: 58-115.
Srinivasan, T. N. (1962b): Review of Sen (1960) in *Journal of Political Economy* 70: 306-307.
Stiglitz, J. E. (1968): "A Note on Technical Choice under Full Employment in a Socialist Economy." *Economic Journal* 78: 603-609.
von Weizsäcker, C. C. (1965): "Existence of Optimal Programs of Accumulation for an Infinite Time Horizon." *Review of Economic Studies* 32: 85-104.
Zaslavsky, A. (2005): "Optimal Programs in the RSS Model." *International Journal of Economic Theory* 1:151-165