

# Spatial competition between shopping centers

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**Abstract.** We study competition between two shopping centers located at the extremes of a linear city, without restricting consumers to make all their purchases at a single place. In the case of competition between a shopping mall and a department store: if the number of goods is low, all the consumers shop at a single place; if it is moderately high, some consumers travel to both extremes to buy each good where it is cheaper (there is a single good that is cheaper at the shopping mall, all the others are cheaper at the department store). The shops at the mall, taken together, obtain a lower profit than the department store. Nevertheless, we expect two shopping malls to endogenously appear in equilibrium.

**Keywords:** Modes of retail, Multi-product firms, Horizontal differentiation, Hotelling model, Two-stop shopping.

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# 1 Introduction

Shopping centers have existed for many centuries as galleries, market squares, bazaars or seaport districts. The oldest indoor space where consumers can buy a huge variety of goods is the *Al-Hamidiyah Souq*, in Damascus (Syria), and dates back to the seventh century. Today, shopping centers are responsible for a very significant fraction of consumer spending. In 2005, the sales revenues of all the shopping centers located in the United States totalled \$1.5 trillion (corresponding to an average of \$5000 *per capita*).<sup>1</sup>

One of the reasons why shopping centers are so attractive is that they allow consumers to buy many kinds of goods without spending much time and money commuting between shops. Therefore, to study competition between shopping centers, one should take into account the demand for many different goods and also the cost of traveling to one or more shopping centers. Most of the existing spatial competition models fail to do so, because they either restrict the analysis to markets with a single good or assume that consumers make all their purchases at the same place (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006). This “one-stop shopping” assumption is very convenient because it allows treating multiple goods as a single bundled good.

We provide a study of competition between shopping centers by extending the standard model of spatial competition (Hotelling, 1929; d’Aspremont, Gabszewicz and Thisse, 1979) to the case of multiple goods without assuming “one-stop shopping”. This extension is straightforward in concept but technically difficult. We consider the existence of two shopping centers located at the extremes of a linear city, selling the same set of goods. Consumers are uniformly spread across the city and buy exactly one unit of each good. They may travel to a shopping center and buy all the goods there or travel to both shopping centers and buy each good where it is cheaper.<sup>2</sup>

A shopping center may be either a shopping mall (where each good is sold by an independent firm) or a department store (where a single firm sells all the goods).<sup>3</sup> We solve for

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<sup>1</sup><http://www.statemaster.com/graph/lif.sho.mal.est.sal-lifestyle-shopping-malls-estimated-sales>.

<sup>2</sup>Consumers are assumed to be fully informed about the prices charged in each extreme of the city.

<sup>3</sup>We rule out bundling strategies (we assume that the price of a bundle of goods is equal to the sum of the price of the individual goods). For an analysis of the bundle pricing problem in a related context, see Armstrong and Vickers (2010). See also Hanson and Martin (1990).

the equilibrium prices, market shares and profits in three scenarios of retail organization: (i) competition between a department store and a shopping mall; (ii) competition between two department stores; (iii) competition between two shopping malls.

In the case of competition between a department store and a shopping mall, we find that there may be consumers visiting the two extremes of the city or not, depending on the number of goods that are sold by the shopping centers. If there are up to four goods, all the consumers make their purchases at a single place (“one-stop shopping”). If there are between seven and eleven goods, some consumers are willing to travel to both extremes of the city to buy each good where it is cheaper (“two-stop shopping”). In this case, there is only one good that is cheaper at the shopping mall than at the department store. However, its price is low enough for some consumers to travel there just to buy this good. If there are five, six or more than eleven goods, there is no price equilibrium in pure strategies.

Regardless of the number of goods, the price of the bundle is lower at the department store than at the shopping mall. This occurs because the otherwise unrelated goods become complements when they are sold at the same location (and substitutes when they are sold at different extremes of the city). When a shop at the mall considers the possibility of decreasing its price, it only cares about the increase of its own demand and not about the increase of the demand of the other shops at the mall. In contrast, the department store internalizes this effect, and takes into account that a decrease in the price of one good also increases the demand for its other goods. In spite of charging a lower price for the bundle, the department store obtains a higher profit than the shops at the mall taken together.

The scenario in which prices are lower is that of competition between two department stores. In this case, the price charged for the bundle of goods is equal to the price charged in the single-good model (independently of the number of goods). The two department stores obviously capture equal shares of the market and obtain equal profits. These are, unsurprisingly, lower than the profits obtained when competing against a shopping mall.

Finally, in the scenario of competition between two shopping malls, we find that each good is sold at the same price as in the single-good model. The shops behave as if consumers only bought their good. This is the competitive scenario in which prices are higher. The explanation is the same as before: the shops at the mall set the same price as in the single-good model because they do not internalize the positive effect of a price decrease on the

other shops at the same mall.

After finding the equilibrium prices in each of the three competitive scenarios, it is straightforward to analyze whether it is more profitable to have a department store offering many products or several independent shops at a mall.<sup>4</sup> We answer this question by considering a two-stage game in which the shopping centers simultaneously choose their modes of retail and then compete in prices. We find that, if the number of goods is not greater than four, it is a dominant strategy to be organized as a shopping mall rather than as a department store. Therefore, the competitive scenario that is expected to appear in equilibrium is that of competition between two shopping malls. However, if the number of goods is between seven and eleven, there is another equilibrium, which is Pareto-inferior: competition between two department stores.

As explained previously, a department store has stronger incentives to charge lower prices than the independent shops at a mall. If the prices of the rival retailers remained the same, the greater aggressiveness of the department store would be profitable. However, setting lower prices induces the rivals to lower their prices as well. If the number of goods is low, this effect dominates, leading to lower profits for everyone. The reason why both sides win when a department store separates into several independent shops was explained by Innes (2006): *“a multi-product retailer can effectively pre-commit to higher prices by organizing itself as a mall of independent outlets”*. If the number of goods is high, it becomes more profitable to compete against a department store by behaving as a department store. But it is still better to compete against a shopping mall by behaving as a shopping mall. This is why the endogenous modes of retail may be two department stores or two shopping malls.

In the literature, the first related result was presented by Edgeworth (1925), who found that it is better, for consumers, to have a single monopolist selling two complementary goods than to have two separate monopolists. Salant, Switzer and Reynolds (1983) came up with a similar result, but in a model of Cournot competition. Using a framework that is closer to ours, Bertrand competition with linear demand, Beggs (1994) concluded that separating into several shops at a mall may be desirable or not, depending on the

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<sup>4</sup>Since otherwise unrelated goods become complements when they are sold at the same shopping center, this question is related to the literature on mergers between firms that sell complementary goods. See, for example, Matutes and Regibeau (1992), Economides and Salop (1992) or Bart (2008).

degree of substitutability between the goods sold at the competing shopping centers (either two department stores or two shopping malls may emerge in equilibrium). Innes (2006) studied the effect of entry and concluded that only department stores survive in equilibrium because they compete more aggressively and, therefore, are more effective in deterring entry. Shopping malls would be driven out of the market by department stores because when there is competition between department stores and shopping malls, the former have higher profits.<sup>5</sup>

We also compare the consumers' surplus and the total surplus in the different competitive scenarios. Since all the consumers are assumed to buy exactly one unit of each good, a change in prices simply transfers surplus between consumers and producers. Therefore, total surplus is maximized when consumers shop at the closest shopping center (transportation costs are minimized). This occurs when there are either two department stores or two shopping malls. Unsurprisingly, the consumers' surplus is the highest in the case of competition between two department stores. Competition between two shopping malls is actually the worst scenario for consumers. In spite of having to support higher transportation costs, consumers are better off when there is a department store and a shopping mall than when there are two shopping malls.

Our model is pioneer in extending the standard spatial competition model (Hotelling, 1929; d'Aspremont, Gabszewicz and Thisse, 1979) to analyze multi-product competition between department stores and shopping malls. To the best of our knowledge, Lal and Matutes (1989) were the first to present a multi-product version of the linear city model of Hotelling (1929). Their objective was to study price discrimination between two types of consumers.<sup>6</sup> Later, Lal and Matutes (1994) and Lal and Rao (1997) introduced imperfect information about prices and the possibility that firms announce the prices that they charge for one or more goods (with advertising acting as a commitment device). In all these works, the analysis is restricted to the case of competition between two department stores that sell two goods. We allow a finite number of goods and an alternative mode of retail: the

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<sup>5</sup>Smith and Hay (2005) have also studied price competition under alternative modes of retail organization (shopping streets, shopping malls and department stores), but did not consider competition between different modes of retail.

<sup>6</sup>In the model of Lal and Matutes (1989), there are two types of consumers: poor and rich. The poor do not support transportation costs. Therefore, they buy each good where it is cheaper ("one-stop shopping" is not assumed). The rich, on the other hand, support transportation costs and, in equilibrium, are not interested in shopping around.

shopping mall.<sup>7</sup>

Other authors have analyzed multi-product price competition, but did not use the spatial competition model to do so. Moreover, most of them based the analysis on the assumption that consumers make all their purchases at the same shopping center (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006).<sup>8</sup> They support this “one-stop shopping” assumption on the fact that shopping implies time and transportation costs. In our opinion, even with the support of empirical works as the one of Rhee and Bell (2002), who have found that consumers make 94% of their weekly groceries expenditures at the same supermarket, it is of interest to relax the assumption that consumers necessarily make all their purchases at the same place.

A duopoly model in which consumers decide whether to buy goods from a single seller or to support an additional cost to buy goods from the two sellers was proposed by Klemperer (1992). He found that if the product lines of the sellers are differentiated, some consumers make “two-stop shopping” to benefit from a greater variety of goods. When the product lines are identical, the motive for “two-stop shopping” disappears. Consumers never make “two-stop shopping” to take advantage of price differences (as they do in our model).<sup>9</sup>

Armstrong and Vickers (2010) also studied competition between two department stores that sell two horizontally differentiated goods, but with consumers located in a square according to their preferences for the different varieties of the two types of goods. In their model, a consumer may prefer the variety of good  $A$  that is sold by one firm and the variety of good  $B$  that is sold by the other firm.<sup>10</sup> For this reason, some consumers make “two-stop

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<sup>7</sup>There are other extensions of the spatial competition model that allow for multi-product firms, but in which consumers only buy one of the goods that are available (Laussel, 2006; Giraud-Heraud, Hammoudi and Mokrane, 2003). Goods available in a shopping center are, in this case, substitutes instead of complements. These models correspond to completely different economic settings.

<sup>8</sup>An exception is the model proposed by Thill (1992) in which a firm that sells two goods competes against a firm that sells only one of the goods. He concluded that some of the consumers that need to buy both goods choose to make “two-stop shopping”.

<sup>9</sup>The structure of the model of Klemperer (1992) implies that consumers always buy equal amounts of every good that is sold by the firms - they buy the whole product line. This prevents the model from capturing a dimension of multi-product competition that is crucial to our results, which is the fact that a firm can sell some goods at a low price to attract consumers and take advantage of commuting costs to be able to sell the other goods at a high price. Given the focus of his contribution, this is perfectly acceptable. The main point of his paper is that the sellers may choose the same product line to decrease the level of competition!

<sup>10</sup>Contrarily to the previously mentioned spatial competition models, in which the goods offered by the firms are assumed to be homogeneous except for the location where they are offered.

shopping”. The contribution of Armstrong and Vickers (2010) has also the notable feature of addressing the bundle pricing problem (they allow the price of the bundle to be lower than the sum of the prices of the individual goods), in contrast with the existing literature on spatial multi-product competition.

Our conclusions show that reducing the research on multi-product competition to the case of two goods may be overly restrictive. In our model, the richness of considering more than two goods has its origin in the following economic mechanism. When there is “two-stop shopping”, the goods are partitioned in two bundles: those that are cheaper at the department store and those that are cheaper at the shopping mall. When the department store decreases the price of the first bundle and increases in the same amount the price of the second, there is an increase in the demand for the first bundle and an equal decrease in the demand for the second bundle. If the first bundle is more expensive, this variation is profitable. Otherwise it is not. This is why the department store sets lower prices than the shopping mall for all the goods except one (maximizes the price of the first bundle and minimizes the price of the second).

The remainder of the article is organized as follows. In Section 2, we setup the model, introduce notation and obtain the demand and the profit functions. In Section 3, we present the possible competitive scenarios and find the equilibrium prices in each one. We study the merger game in Section 4. Section 5 is dedicated to a welfare analysis. Section 6 concludes the article with some remarks. Most of the proofs are collected in the Appendix.

## 2 The model

### 2.1 Basic setup

We consider a multi-product version of the model of Hotelling (1929). There is a continuum of consumers uniformly distributed across a linear city,  $[0, 1]$ . Each consumer buys one unit of each of the products,  $i \in \mathcal{I} \equiv \{1, \dots, n\}$ , which are sold at the extremes of the city ( $x = 0$  and  $x = 1$ ). The price of good  $i$  at the left extreme ( $L$ ) is denoted by  $p_{iL}$  and the price of good  $i$  at the right extreme ( $R$ ) is denoted by  $p_{iR}$ .

The reservation price for each product,  $V_i$ , is assumed to be high enough for the market to be fully covered. Thus, the demand is perfectly inelastic and the only decision of consumers is where to buy each product. Each consumer chooses among three possibilities:

( $L$ ) to buy all the goods at  $x = 0$ ;

( $R$ ) to buy all the goods at  $x = 1$ ;

( $LR$ ) to travel to both extremes and buy each good where it is cheaper.

We denote by  $P_L$  and by  $P_R$  the price that a consumer pays for all the goods at  $x = 0$  and at  $x = 1$ , respectively ( $P_L = \sum_{i=1}^n p_{iL}$  and  $P_R = \sum_{i=1}^n p_{iR}$ ). By  $P_{LR}$ , we denote the price that a consumer pays for all the goods if she buys each good where it is cheaper ( $P_{LR} = \sum_{i=1}^n \min\{p_{iL}, p_{iR}\}$ ).

To make their decision, consumers take into account not only the prices charged for the products, but also the transportation costs that they must support to acquire them. We assume that the transportation costs are linear in distance. Let  $u_L(x)$ ,  $u_R(x)$  and  $u_{LR}(x)$  denote the utility attained by an agent located at  $x \in [0, 1]$  who chooses to purchase, respectively: ( $L$ ) all the goods at  $x = 0$ ; ( $R$ ) all the goods at  $x = 1$ ; ( $LR$ ) each good where it is cheaper. Then:

$$\begin{aligned} u_L(x) &= \sum_{i=1}^n V_i - P_L - tx, \\ u_R(x) &= \sum_{i=1}^n V_i - P_R - t(1 - x), \\ u_{LR}(x) &= \sum_{i=1}^n V_i - P_{LR} - t. \end{aligned}$$

It is important to keep in mind that if a consumer travels to both extremes, she supports higher transportation costs than if she had chosen to purchase all the goods at the same location. For this reason, the demand for each product at a certain location is related to the demand for any other product at any location. Products sold at the same location are complementary goods, while products sold at different locations are substitutes.

## 2.2 Demand and profit functions

The consumers that are most likely to purchase a good that is sold at one of the extremes are those who are located closer to that extreme. When all the goods have strictly positive demand at both locations, the consumers near the left extreme are surely buying all the goods at  $x = 0$  (their choice is  $L$ ), while those near the right extreme are surely buying all

the goods at  $x = 1$  (their choice is  $R$ ).

Depending on the prices charged for each good at each location, some consumers may find it worthwhile to travel to both extremes of the city, to buy each good where it is cheaper. This occurs if some goods are sufficiently cheaper at  $x = 0$  while other goods are sufficiently cheaper at  $x = 1$ . On the contrary, if the price differences across locations are relatively small, then all the consumers make their purchases at a single location, either at  $x = 0$  or at  $x = 1$ . These possible demand scenarios (one-stop shopping and two-stop shopping) are illustrated in Figure 1.

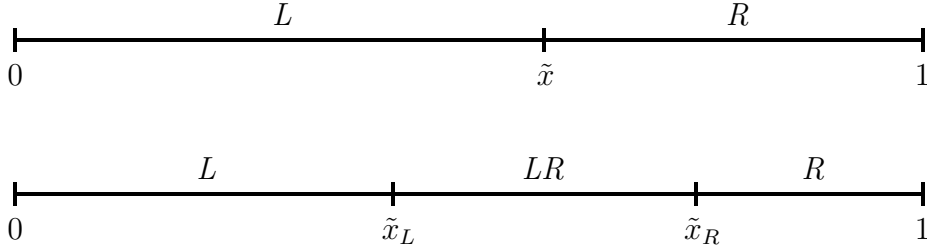


Figure 1: Possible demand scenarios.

To obtain the demand for each good at each location, it is useful to find the location of the consumer that is indifferent between each pair of choices (among  $L$ ,  $R$  and  $LR$ ). Accordingly, we use some additional notation.

By  $\tilde{x}_L$ , we denote the location of the consumer that is indifferent between  $L$  and  $LR$ :

$$u_L(\tilde{x}_L) = u_{LR}(\tilde{x}_L) \Leftrightarrow \tilde{x}_L = 1 - \frac{P_L - P_{LR}}{t}.$$

We denote by  $\tilde{x}_R$  the consumer that is indifferent between  $R$  and  $LR$ :

$$u_R(\tilde{x}_R) = u_{LR}(\tilde{x}_R) \Leftrightarrow \tilde{x}_R = \frac{P_R - P_{LR}}{t}.$$

Finally, we denote by  $\tilde{x}$  the consumer that is indifferent between  $L$  and  $R$ . It is clear from the expression below that  $\tilde{x} = \frac{\tilde{x}_L + \tilde{x}_R}{2}$ :

$$u_L(\tilde{x}) = u_R(\tilde{x}) \Leftrightarrow \tilde{x} = \frac{1}{2} + \frac{P_R - P_L}{2t}.$$

There are consumers traveling to both extremes of the city if  $\tilde{x}_L < \tilde{x}_R$ , which is equivalent to  $\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| > t$ . Otherwise, all the consumers make their purchases at a single place. It is easy to verify that  $\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq t$  implies that  $0 \leq \tilde{x} \leq 1$ . Therefore, in this case, the demand for each good sold at  $L$  is  $\tilde{x}$  and the demand for each good sold at  $R$  is  $1 - \tilde{x}$ .

It is convenient to denote the vector of prices of all the goods at both locations by  $\mathbf{p} \in \mathbb{R}_+^{2n}$  and to consider the following sets:

$$\begin{aligned}\mathcal{P}_1 &= \{\mathbf{p} \in \mathbb{R}_+^{2n} : \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq t\}; \\ \mathcal{P}_2 &= \{\mathbf{p} \in \mathbb{R}_+^{2n} : \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| > t\}.\end{aligned}$$

If there are consumers that travel to both extremes, the demand for a good depends on whether this good is cheaper at  $L$  or at  $R$ . Denoting by  $\mathcal{I}_L$  and  $\mathcal{I}_R$  the sets of goods that are strictly cheaper at  $L$  and  $R$ , respectively, we can write the expressions for the indifferent consumers as follows:

$$\tilde{x}_L = 1 - \frac{1}{t} \sum_{i \in \mathcal{I}_R} (p_{iL} - p_{iR})$$

and

$$\tilde{x}_R = \frac{1}{t} \sum_{i \in \mathcal{I}_L} (p_{iR} - p_{iL}).$$

The demand for a good  $i \in \mathcal{I}_L$  at  $L$  is  $\min\{\tilde{x}_R, 1\}$ , while its demand at  $R$  is  $\max\{0, 1 - \tilde{x}_R\}$ . If  $i \in \mathcal{I}_R$ , its demand at  $L$  is  $\max\{0, \tilde{x}_L\}$  and its demand at  $R$  is  $\min\{1 - \tilde{x}_L, 1\}$ . In case of a tie ( $p_{iL} = p_{iR}$ ), each consumer that travels to both extremes may either buy good  $i$  at  $L$  or at  $R$ . Any tie-breaking assumption leads to the same results. We can assume, for example, that half of the consumers buys good  $i$  at  $L$  and the other half buys it at  $R$ .

The demand for good  $i$  at  $x = 0$  is:

$$q_{iL} = \begin{cases} \tilde{x} & \text{if } \mathbf{p} \in \mathcal{P}_1 \\ \min\{\tilde{x}_R, 1\} & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iL} < p_{iR} \\ \frac{1}{2}(\min\{\tilde{x}_R, 1\} + \max\{0, \tilde{x}_L\}) & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iL} = p_{iR} \\ \max\{0, \tilde{x}_L\} & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iL} > p_{iR} \end{cases},$$

while the demand for the same good at  $x = 1$  is  $q_{iR} = 1 - q_{iL}$ .

The marginal cost of producing one unit of each of the goods is assumed to be zero. Under this assumption, the profits coincide with the sales revenues. This simplification does not affect any of the results in the paper.<sup>11</sup>

The profit that results from selling good  $i$  at  $x = 0$  is:

$$\Pi_{iL} = \begin{cases} p_{iL} \left( \frac{1}{2} + \frac{P_R - P_L}{2t} \right) & \text{if } \mathbf{p} \in \mathcal{P}_1 \\ p_{iL} \min \left\{ \frac{P_R - P_{LR}}{t}, 1 \right\} & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iL} < p_{iR} \\ \frac{p_{iL}}{2} \left( \min \left\{ \frac{P_R - P_{LR}}{t}, 1 \right\} + \max \left\{ 0, 1 - \frac{P_L - P_{LR}}{t} \right\} \right) & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iL} = p_{iR} \\ p_{iL} \max \left\{ 0, 1 - \frac{P_L - P_{LR}}{t} \right\} & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iL} > p_{iR} \end{cases}.$$

By symmetry, the profit that results from selling good  $i$  at the right extreme of the city is:

$$\Pi_{iR} = \begin{cases} p_{iR} \left( \frac{1}{2} + \frac{P_L - P_R}{2t} \right) & \text{if } \mathbf{p} \in \mathcal{P}_1 \\ p_{iR} \min \left\{ \frac{P_L - P_{LR}}{t}, 1 \right\} & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iR} < p_{iL} \\ \frac{p_{iR}}{2} \left( \min \left\{ \frac{P_L - P_{LR}}{t}, 1 \right\} + \max \left\{ 0, 1 - \frac{P_R - P_{LR}}{t} \right\} \right) & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iR} = p_{iL} \\ p_{iR} \max \left\{ 0, 1 - \frac{P_R - P_{LR}}{t} \right\} & \text{if } \mathbf{p} \in \mathcal{P}_2 \wedge p_{iR} > p_{iL} \end{cases}.$$

### 2.3 Modes of retail and price-setting behavior

On the supply side, we consider two different modes of retail: department stores and shopping malls. A department store is a multi-product firm that sells the  $n$  goods at the same location. For example, a department store at  $x = 0$  sells goods  $\{iL\}_{i \in \mathcal{I}}$ , seeking to maximize its profit,  $\Pi_L = \sum_{i=1}^n \Pi_{iL}$ . A shopping mall is a group of single-product firms that sell each of the  $n$  goods at the same location. For example, a shopping mall at  $x = 1$  is composed by  $n$  firms, each firm selling one good,  $iR$ , with the objective of maximizing its individual profit,  $\Pi_{iR}$ . We exclude the possibility of coordinated behavior among shops at a mall. Each shop chooses how much to charge for the product it sells, taking the remaining prices as given.

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<sup>11</sup>To obtain the equilibrium prices for the case in which the marginal costs are constant but different from zero, simply add the marginal cost to the equilibrium prices that we obtain.

### 2.3.1 Profit maximization by a department store

With prices that induce one-stop shopping (which are such that  $\mathbf{p} \in \mathcal{P}_1$ ), the profit of a department store is:

$$\Pi_L = P_L \left( \frac{1}{2} + \frac{P_R - P_L}{2t} \right).$$

In this domain, profit maximization only requires that:

$$P_L = \frac{P_R}{2} + \frac{t}{2}. \quad (1)$$

It is always possible for the department store to set prices that add up to this  $P_L$  and that induce one-stop shopping. For example, by setting  $p_{iL} = \frac{P_L}{P_R} p_{iR}, \forall i \in \mathcal{I}$ .

With prices that induce two-stop shopping (which are such that  $\mathbf{p} \in \mathcal{P}_2$ ), the profit of the department store is given by:

$$\Pi_L = \frac{1}{t} \sum_{i \in \mathcal{I}_L} p_{iL} \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}) + \sum_{i \in \mathcal{I}_R} p_{iL} \left[ 1 - \frac{1}{t} \sum_{j \in \mathcal{I}_R} (p_{jL} - p_{jR}) \right]. \quad (2)$$

For any  $i \in \mathcal{I}_L$ , the first-order condition is:

$$\frac{\partial \Pi_L}{\partial p_{iL}} = 0 \Leftrightarrow \sum_{i \in \mathcal{I}_L} p_{iL} = \frac{1}{2} \sum_{i \in \mathcal{I}_L} p_{iR}. \quad (3)$$

While for any  $i \in \mathcal{I}_R$ , the first-order condition is:

$$\frac{\partial \Pi_L}{\partial p_{iL}} = 0 \Leftrightarrow \sum_{i \in \mathcal{I}_R} p_{iL} = \frac{1}{2} \sum_{i \in \mathcal{I}_R} p_{iR} + \frac{t}{2}. \quad (4)$$

The prices that satisfy these first-order conditions induce two-stop shopping (that is, imply that  $\mathbf{p} \in \mathcal{P}_2$ ) if and only if:

$$\sum_{i \in \mathcal{I}_L} p_{iR} - \sum_{i \in \mathcal{I}_R} p_{iR} > t. \quad (5)$$

This is actually a necessary and sufficient condition for the department store to be interested in inducing two-stop shopping.

**Lemma 1.** *A department store located at  $x = 0$  prefers to induce two-stop shopping if and only if:  $\sum_{i \in \mathcal{I}_L} p_{iR} - \sum_{i \in \mathcal{I}_R} p_{iR} > t$ .*

*Proof.* Observe that if the prices at the right extreme do not satisfy condition (5), then, to induce two-stop shopping, the department store would have to sell the bundle of goods  $\mathcal{I}_L$  at a lower price than the bundle  $\mathcal{I}_R$ :

$$\begin{cases} \sum_{i \in \mathcal{I}_L} p_{iR} - \sum_{i \in \mathcal{I}_R} p_{iR} \leq t \\ \sum_{i \in \mathcal{I}_L} (p_{iR} - p_{iL}) + \sum_{i \in \mathcal{I}_R} (p_{iL} - p_{iR}) > t \end{cases} \Rightarrow \sum_{i \in \mathcal{I}_L} p_{iL} < \sum_{i \in \mathcal{I}_R} p_{iL}.$$

But this implies that, for any given  $P_L$ , the one-stop shopping scenario is more profitable than the two-stop shopping scenario:

$$\begin{aligned} \sum_{i \in \mathcal{I}_L} p_{iL} < \sum_{i \in \mathcal{I}_R} p_{iL} &\Leftrightarrow \left( \tilde{x}_R - \frac{\tilde{x}_R}{2} - \tilde{x}_L \right) \sum_{i \in \mathcal{I}_L} p_{iL} + \left( \tilde{x}_L - \frac{\tilde{x}_L}{2} - \tilde{x}_R \right) \sum_{i \in \mathcal{I}_R} p_{iL} < 0 \Leftrightarrow \\ &\Leftrightarrow \tilde{x}_R \sum_{i \in \mathcal{I}_L} p_{iL} + \tilde{x}_L \sum_{i \in \mathcal{I}_R} p_{iL} < \tilde{x} P_L. \end{aligned} \quad (6)$$

□

Observe that adding (3) and (4), we obtain (1). This means that the department store actually charges the same price for the bundle of  $n$  goods regardless of whether it induces two-stop shopping or not.

**Lemma 2.** *A department store located at  $x = 0$  always sets the price of the bundle of  $n$  goods to:  $P_L = \frac{P_R}{2} + \frac{t}{2}$ .*

When a department store induces two-stop shopping, only one of the goods is more expensive there than at the other shopping center. All the other goods are cheaper.

**Lemma 3.** *When inducing two-stop shopping, a department store located at  $x = 0$  typically chooses the set  $\mathcal{I}_R$  to contain a single element,  $j \in \operatorname{argmin}_{i \in \mathcal{I}} \{p_{iR}\}$ . The only exception is when there is more than one good  $i$  with  $p_{iR} = 0$ . In this case,  $\mathcal{I}_R$  can be any non-empty subset of those goods.*

*Proof.* See the Appendix. □

To understand this result, it may be useful to look at two-stop shopping as a situation in which there is competition in markets for two bundled goods:  $\mathcal{I}_L$  and  $\mathcal{I}_R$ . The department store faces the problem of choosing the partition of  $\mathcal{I}$  that defines the two bundles. Suppose that the prices are initially the same for all the goods and at both locations. When the department store decreases the price of a bundle of goods and increases the price of the remaining goods maintaining the total price constant, there is an increase in the demand for the first bundle and an equal decrease in the demand for the second bundle. The more expensive is the first bundle and the cheaper is the second bundle, the more profitable is this variation. This is why the department store sets lower prices than the shopping mall for all the goods except one.

The partition that is chosen maximizes the price of  $\mathcal{I}_L$  at the shopping mall and minimizes the price of  $\mathcal{I}_R$  at the shopping mall. The department store, then, sells  $\mathcal{I}_L$  at a lower price and  $\mathcal{I}_R$  at a higher price. In the end, two-stop shopping is profitable if the department store is able to sell  $\mathcal{I}_L$  (the bundle for which it has a high demand) at a higher price than  $\mathcal{I}_R$  (the bundle for which it has a low demand).

The profit-maximizing behavior of a department store is summarized in Proposition 1.

**Proposition 1.** *Let  $j \in \operatorname{argmin}_{i \in \mathcal{I}} \{p_{iR}\}$ . If  $\sum_{i \neq j} p_{iR} - p_{jR} \leq t$ , a department store located at  $x = 0$  induces one-stop shopping, setting prices for the individual goods that are such that:  $\sum_i |p_{iL} - p_{iR}| \leq t$  and  $P_L = \frac{P_R}{2} + \frac{t}{2}$ . Otherwise, the department store induces two-stop shopping, setting  $\sum_{i \neq j} p_{iL} = \frac{1}{2} \sum_{i \neq j} p_{iR}$  (with  $p_{iL} < p_{iR}, \forall i \neq j$ ) and  $p_{jL} = \frac{1}{2} p_{jR} + \frac{t}{2}$ .*

### 2.3.2 Profit maximization by the shops at the mall

In this subsection, we consider the profit-maximization problem of an individual shop located at the right extreme of the city.

To study the behavior of  $\Pi_{iR}$  as a function of  $p_{iR}$ , it is convenient to define a partition of the domain of  $p_{iR}$  that separates the cases in which: ( $D_1$ ) all consumers buy good  $i$  at  $R$ ; ( $D_2$ ) there is two-stop shopping with  $i \in \mathcal{I}_R$ ; ( $D_3$ ) there is one-stop shopping; ( $D_2$ )

there is two-stop shopping with  $i \in \mathcal{I}_L$ ; ( $D_5$ ) no consumer buys good  $i$  at  $R$ :

$$\left\{ \begin{array}{l} D_1 = [0, -t + p_{iL} + s_{Ri}] \\ D_2 = ]-t + p_{iL} + s_{Ri}, -t + p_{iL} + s_{Li} + s_{Ri}[ \\ D_3 = [-t + p_{iL} + s_{Li} + s_{Ri}, t + p_{iL} - s_{Li} - s_{Ri}] \\ D_4 = ]t + p_{iL} - s_{Li} - s_{Ri}, t + p_{iL} - s_{Li}[ \\ D_5 = [t + p_{iL} - s_{Li}, +\infty[ , \end{array} \right.$$

where  $s_{Li} = \sum_{j \in \mathcal{I}_L \setminus \{i\}} (p_{jR} - p_{jL})$  and  $s_{Ri} = \sum_{j \in \mathcal{I}_R \setminus \{i\}} (p_{jL} - p_{jR})$ . The partition above is valid as long as  $s_{Li} + s_{Ri} \leq t$ , which implies that there may be one-stop shopping or two-stop shopping depending on the value of  $p_{iR}$ . Otherwise,  $D_3$  becomes empty and the transition between  $D_2$  and  $D_4$  occurs at  $p_{iR} = p_{iL}$ .

Accordingly, the demand for good  $iR$ , as a function of  $p_{iR}$ , is:

$$q_{iR} = \begin{cases} 1, & p_{iR} \in D_1 \\ \frac{1}{t} \sum_{j \in \mathcal{I}_R} (p_{jL} - p_{jR}), & p_{iR} \in D_2 \\ \frac{1}{2} + \frac{1}{2t} P_L - \frac{1}{2t} P_R, & p_{iR} \in D_3 \\ 1 - \frac{1}{t} \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}), & p_{iR} \in D_4 \\ 0, & p_{iR} \in D_5 \end{cases} . \quad (7)$$

The demand is linear in each branch. Its derivative is initially zero (in  $D_1$ ), then it is  $-\frac{1}{t}$  (in  $D_2$ ), changes to  $-\frac{1}{2t}$  (in  $D_3$ ), becomes  $-\frac{1}{t}$  again (in  $D_4$ ) and, finally, vanishes (in  $D_5$ ). Accordingly, the profit function is concave in each branch. It starts at zero (for  $p_{iR} = 0$ ) and ends at zero (for  $p_{iR} \in D_5$ ).

If  $D_3$  is not empty ( $s_{Li} + s_{Ri} \leq t$ ), the demand and the profit are globally continuous. Otherwise ( $s_{Li} + s_{Ri} > t$ ), they jump downwards at the transition between  $D_2$  and  $D_4$ .

In this subsection, we will not provide a complete characterization of the price-setting behavior of the shops at a mall. We only write, for future reference, the profit function in the relevant branches ( $D_2$ ,  $D_3$  and  $D_4$ ) and the corresponding first-order conditions.

In  $D_2$ , the profit of the shop is:

$$\Pi_{iR} = \frac{1}{t} p_{iR} \sum_{j \in \mathcal{I}_R} (p_{jL} - p_{jR}),$$

which leads to the following first-order condition:

$$p_{iR} = \sum_{j \in \mathcal{I}_R} (p_{jL} - p_{jR}) \Leftrightarrow p_{iR} = \frac{1}{2} \sum_{j \in \mathcal{I}_R} p_{jL} - \frac{1}{2} \sum_{j \in \mathcal{I}_R \setminus \{i\}} p_{jR}. \quad (8)$$

In  $D_3$ , the profit of the shop is:

$$\Pi_{iR} = p_{iR} \left( \frac{1}{2} + \frac{P_L - P_R}{2t} \right).$$

The corresponding first-order condition is:

$$p_{iR} = P_L - P_R + t \Leftrightarrow p_{iR} = \frac{P_L}{2} - \frac{1}{2} \sum_{j \in \mathcal{I} \setminus \{i\}} p_{jR} + \frac{t}{2}. \quad (9)$$

Finally, in  $D_4$ , the profit of the shop is:

$$\Pi_{iR} = \frac{p_{iR}}{t} \left[ t - \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}) \right],$$

and the first-order condition is given by:

$$p_{iR} = t - \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}) \Leftrightarrow p_{iR} = \frac{t}{2} + \frac{1}{2} \sum_{j \in \mathcal{I}_L} p_{jL} - \frac{1}{2} \sum_{j \in \mathcal{I}_L \setminus \{i\}} p_{jR}. \quad (10)$$

### 3 Competitive scenarios

In this section, we study the following competitive scenarios:

- a department store at  $x = 0$  and a shopping mall at  $x = 1$ ;
- two department stores, one at  $x = 0$  and another at  $x = 1$ ;
- two shopping malls, one at  $x = 0$  and another at  $x = 1$ .

### 3.1 Competition between a department store and a shopping mall

We start by considering the case in which there is a department store located at  $x = 0$  and a shopping mall located at  $x = 1$ . The department store chooses the prices of the  $n$  goods with the objective of maximizing its total profit ( $\sum_{i=1}^n \Pi_{iL}$ ), while each of the shops at the mall seeks to maximize its individual profit ( $\Pi_{iR}$ ).

#### 3.1.1 Equilibria with one-stop shopping

In an equilibrium with one-stop shopping, the first-order conditions for profit-maximization by the shops at the mall (9) imply that:

$$p_{iR} = P_L - P_R + t \Rightarrow P_R = np_{iR} = nP_L - nP_R + nt \Rightarrow P_R = \frac{n}{n+1} (P_L + t). \quad (11)$$

Combining this condition with the first-order condition for profit-maximization by the department store (1), we obtain the candidate equilibrium prices (see Figure 2):

$$\begin{cases} P_L = \frac{P_R}{2} + \frac{t}{2} \\ P_R = \frac{n}{n+1} (P_L + t) \end{cases} \Rightarrow \begin{cases} P_L = \frac{2n+1}{n+2} t \\ P_R = \frac{3n}{n+2} t \end{cases} \text{ with } p_{iR} = \frac{3}{n+2} t, \forall i \in \mathcal{I}. \quad (12)$$

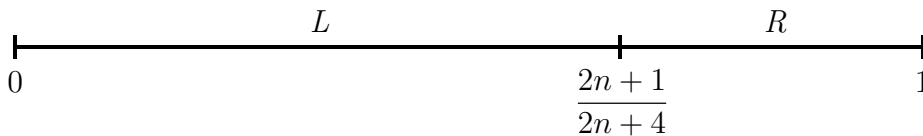


Figure 2: Candidate equilibrium with one-stop shopping.

By Lemmas 1 and 3, we know that the department store prefers to deviate and set prices that induce two-stop shopping if and only if:

$$\sum_{i \in \mathcal{I}_L} p_{iR} - \sum_{i \in \mathcal{I}_R} p_{iR} > t \Leftrightarrow \frac{3(n-2)}{n+2} t > t \Leftrightarrow n > 4.$$

This means that the above situation (12) can only be an equilibrium for  $n \leq 4$ .

To better understand why the equilibrium with one-stop shopping requires that  $n \leq 4$ , see the comparison (for  $n = 5$ ) between the equilibrium candidate and the profit-maximizing deviation by the department store in Figure 3.

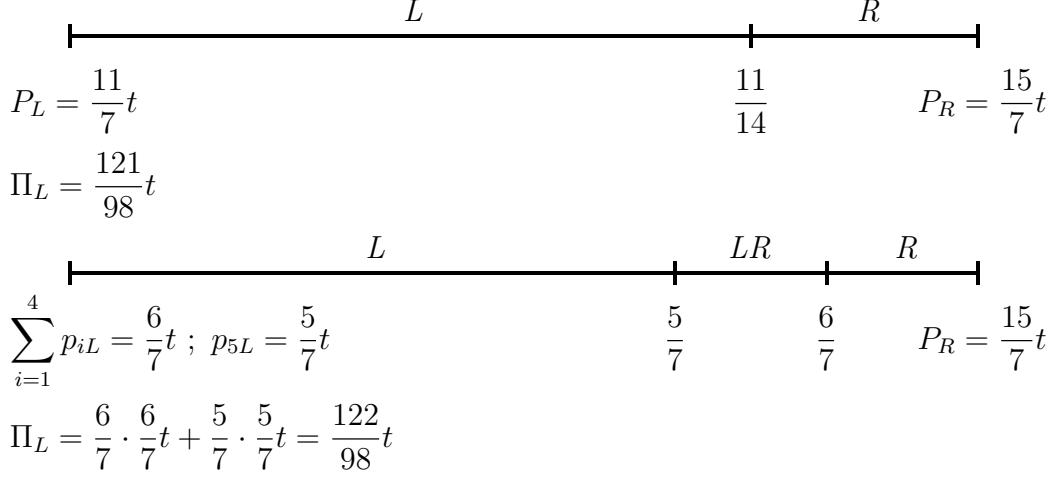


Figure 3: Comparison between the candidate one-stop shopping equilibrium (top) and the optimal deviation by the department store (bottom) when  $n = 5$ .

To avoid that the shops at the mall deviate to a situation with two-stop shopping, we need to impose the following condition on the individual prices at the department store:<sup>12</sup>

$$\sum_{i=1}^n \left| p_{iL} - \frac{3}{n+2}t \right| \leq \frac{n+6\sqrt{2}-7}{n+2}t < t.$$

**Proposition 2.** *In the case of competition between a department store and a shopping mall, there is an equilibrium with one-stop shopping if and only if  $n \leq 4$ . It is such that:*

(1) *It is cheaper to buy the  $n$  goods at the department store than at the shopping mall:*

$$\begin{cases} P_L = \sum_{i=1}^n p_{iL} = \frac{2n+1}{n+2}t, & \text{with } \sum_{i=1}^n \left| p_{iL} - \frac{3}{n+2}t \right| \leq \frac{n+6\sqrt{2}-7}{n+2}t < t \\ P_R = \sum_{i=1}^n p_{iR} = \frac{3n}{n+2}t, & \text{with } p_{iR} = \frac{3}{n+2}t, \forall i \in \mathcal{I} \end{cases}.$$

<sup>12</sup>This is shown in the proof of Proposition 2. See the Appendix.

(2) *The demand is greater at the department store:*

$$\begin{cases} q_{iL} = \frac{2n+1}{2n+4} \\ q_{iR} = \frac{3}{2n+4} \end{cases} .$$

(3) *The department store earns more profits than the shops at the mall taken together:*

$$\begin{cases} \Pi_L = \sum_{i=1}^n \Pi_{iL} = \frac{(2n+1)^2}{2(n+2)^2} t \\ \Pi_R = \sum_{i=1}^n \Pi_{iR} = \frac{9n}{2(n+2)^2} t \end{cases} .$$

*Proof.* See the Appendix. □

The department store does not care about how much to charge for each individual good because all its customers buy the entire bundle of goods. What matters for the department store is the price of the bundle.<sup>13</sup>

The department store charges a lower price for the bundle of  $n$  goods because, when compared with the shops at the mall, it has an additional incentive to set low prices. By decreasing the price of one good (for example, the price of books), the department store increases the demand for all the goods that are sold there (books, groceries, etc.). At the shopping mall, the bookshop, when choosing the price to set for books, only takes into account the effect on its own demand, ignoring the effect of the price of books on the demand for groceries and for the remaining goods.

As a result of setting lower prices, the department store captures more than half of the market. It does not capture the whole market because the customers that are closer to the shopping mall weight the price advantage of the department store against the proximity advantage of the shopping mall. In equilibrium, the shopping mall retains the consumers that are sufficiently close.

Comparing the joint profit at each extreme of the city, we find that the department store earns more than the shops at the mall taken together.

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<sup>13</sup>The indeterminacy of individual prices at the department store does not extend to the demand.

### 3.1.2 Equilibria with two-stop shopping

In an equilibrium with two-stop shopping, the department store must be selling  $n - 1$  goods at a lower price than the shopping mall and a single good at a higher price than the shopping mall (see Lemma 3).

Adding the first-order conditions for the profit-maximization by the  $n - 1$  shops at the mall that sell goods in  $\mathcal{I}_L$ , given by (10), we obtain the following reaction function:

$$\sum_{i \in \mathcal{I}_L} p_{iR} = (n - 1)t - (n - 1) \sum_{i \in \mathcal{I}_L} (p_{iR} - p_{iL}) \Leftrightarrow \sum_{i \in \mathcal{I}_L} p_{iR} = \frac{n - 1}{n} \left( t + \sum_{i \in \mathcal{I}_L} p_{iL} \right).$$

Combining it with the first-order condition for profit-maximization by the department store (3), we obtain:

$$\begin{cases} \sum_{i \in \mathcal{I}_L} p_{iR} = \frac{2n-2}{n+1}t, & \text{with } p_{iR} = \frac{2}{n+1}t, \forall i \in \mathcal{I}_L \\ \sum_{i \in \mathcal{I}_L} p_{iL} = \frac{n-1}{n+1}t \end{cases}.$$

For the single shop at the mall that sells a good  $i \in \mathcal{I}_R$ , the first-order condition for profit-maximization is (8):

$$p_{iR} = \frac{p_{iL}}{2}.$$

While the corresponding first-order condition for profit-maximization by the department store is (4):

$$p_{iL} = \frac{p_{iR}}{2} + \frac{t}{2}.$$

Combining the two conditions, we obtain:

$$p_{iR} = \frac{t}{3} \quad \text{and} \quad p_{iL} = \frac{2t}{3}.$$

In spite of charging a higher price, the department store has a greater demand because many of its customers do not find it profitable to make two-stop shopping:

$$q_{iR} = \frac{1}{3} \quad \text{and} \quad q_{iL} = \frac{2}{3}.$$

The candidate equilibrium with two-stop shopping is represented in Figure 4.



Figure 4: Candidate equilibrium with two-stop shopping.

The department store is only interested in setting prices that induce two-stop shopping if condition (5) is satisfied. Substituting the expressions for the candidate equilibrium prices at  $x = 1$ , this condition becomes:

$$\frac{2n-2}{n+1}t - \frac{t}{3} > t \Leftrightarrow n > 5.$$

We must also verify that the shop at the mall that is setting the low price does not deviate to a price that induces one-stop shopping. From, the first-order condition (9), we find the following candidate deviation (which induces one-stop shopping for  $n \leq 8$ ):

$$p_{iR} = \frac{P_L}{2} - \frac{1}{2} \sum_{j \in \mathcal{I} \setminus \{i\}} p_{jR} + \frac{t}{2} = \frac{n-1}{2(n+1)}t + \frac{t}{3} - \frac{n-1}{n+1}t + \frac{t}{2} = \frac{n+4}{3(n+1)}t.$$

The corresponding profit is:

$$\Pi_{iR} = p_{iR} (1 - \tilde{x}) = \frac{(n+4)^2}{18(n+1)^2}t.$$

It is higher than the equilibrium profit when:

$$\frac{(n+4)^2}{18(n+1)^2}t > \frac{1}{9}t \Leftrightarrow (n+4)^2 > 2(n+1)^2 \Leftrightarrow n < 6.243.$$

The profit function of this shop (with all the other prices at their candidate equilibrium levels), for  $n = 6$  and  $n = 7$ , is shown in Figure 5.

Finally, we need to verify that the shops at the mall that are setting higher prices than the department store do not deviate to a price that induces one-stop shopping. In the Appendix we show that they deviate from the candidate equilibrium when  $n \geq 12$ .

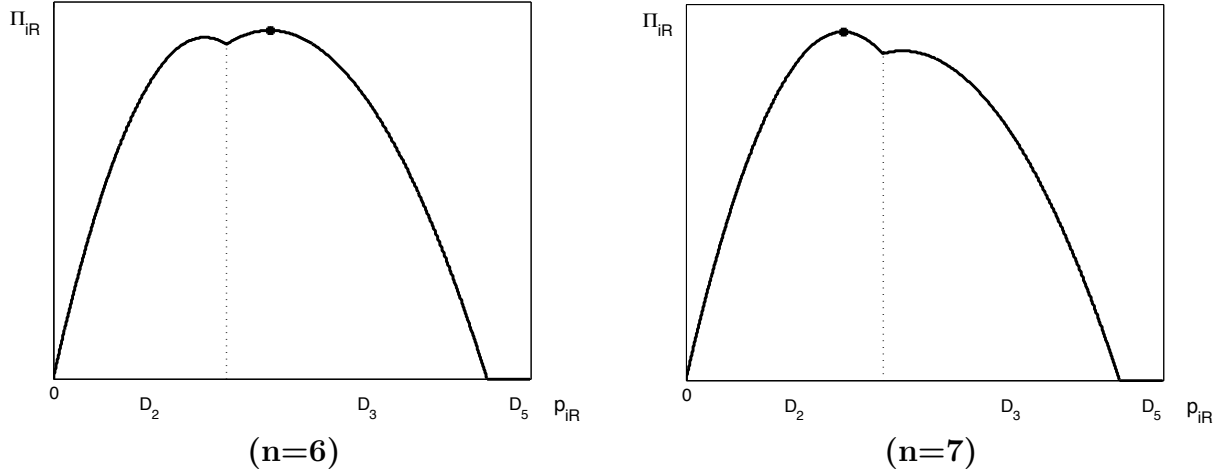


Figure 5: Profit function of the shop that sells the cheap good at the mall, given that the other stores are charging the prices of the candidate equilibrium with two-stop shopping.

As illustrated in Figure 6, for  $n \geq 12$  it is preferable for these shops to decrease prices with the objective of capturing the customers that make two-stop shopping.

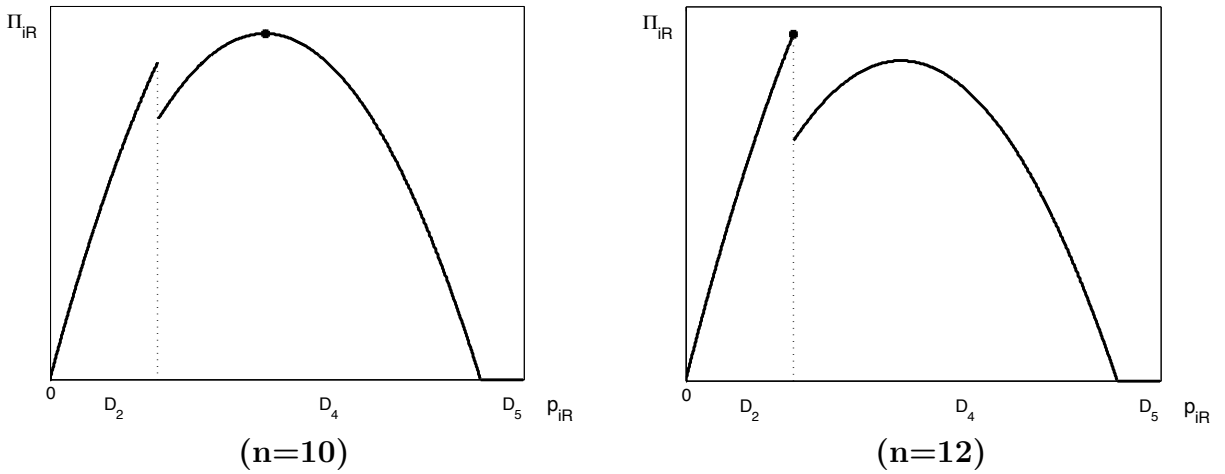


Figure 6: Profit function of a shop that sells an expensive good at the mall, given that the other stores are charging the prices of the candidate equilibrium with two-stop shopping.

**Proposition 3.** *In the case of competition between a department store and a shopping mall, there is an equilibrium with two-stop shopping if and only if  $7 \leq n \leq 11$ . It is such that:*

- (1) *It is cheaper to buy  $n - 1$  of the  $n$  goods at the department store than at the shopping mall:  $\#\mathcal{I}_L = n - 1$  and  $\#\mathcal{I}_R = 1$ .*

(2) The prices of the goods that are cheaper at the department store,  $i \in \mathcal{I}_L$ , are such that:

$$\begin{cases} \sum_{i \in \mathcal{I}_L} p_{iL} = \frac{n-1}{n+1}t, & \text{with } p_{iL} \leq \frac{12}{(n+1)^2}t, \forall i \in \mathcal{I}_L \\ \sum_{i \in \mathcal{I}_L} p_{iR} = \frac{2(n-1)}{n+1}t, & \text{with } p_{iR} = \frac{2}{n+1}t, \forall i \in \mathcal{I}_L \end{cases},$$

and the corresponding demands are:

$$q_{iL} = \frac{n-1}{n+1} \quad \text{and} \quad q_{iR} = \frac{2}{n+1}.$$

(3) The prices of the only good that is cheaper at the shopping mall,  $i \in \mathcal{I}_R$ , are:

$$p_{iL} = \frac{2}{3}t \quad \text{and} \quad p_{iR} = \frac{1}{3}t,$$

and the demands are:

$$q_{iL} = \frac{2}{3} \quad \text{and} \quad q_{iR} = \frac{1}{3}.$$

(4) The department store earns more profits than the shops at the mall taken together:

$$\begin{cases} \Pi_L = \frac{(n-1)^2}{(n+1)^2}t + \frac{4t}{9} \\ \Pi_R = \frac{4(n-1)}{(n+1)^2}t + \frac{t}{9} \end{cases}.$$

Curiously, independently of the number of goods, the consumers that buy all the products at the department store are those located at  $[0, \frac{2}{3}[$ . For a consumer located at  $x \in ]\frac{2}{3}, 1]$  who visits the department store to buy the goods  $i \in \mathcal{I}_L$ , the extra transportation cost of also visiting the shopping mall is smaller than  $\frac{t}{3}$ . However, the difference in the price of good  $i \in \mathcal{I}_R$  is:  $p_{iL} - p_{iR} = \frac{t}{3}$ . Thus, for all these consumers, it is worthwhile to buy the good  $i \in \mathcal{I}_R$  at the shopping mall. As the number of goods increases, there are more consumers willing to make their purchases at both extremes of the city.

Again, as in the equilibria with one-stop shopping, the department store has a higher profit than all the shops at the mall taken together.

When the number of goods is low ( $n \leq 6$ ), the shop that sells the cheap good at the mall is not capturing (in the candidate equilibrium) much more customers than the shops that sell the expensive goods (see Figure 4). It prefers to deviate and set a higher price,

that induces one-stop shopping. As the number of goods increases, to sell a cheap good becomes more profitable (the difference between  $\frac{n-1}{n+1}$  and  $\frac{2}{3}$  increases). When the number of goods is high ( $n \geq 12$ ), the shops that sell the expensive goods at the mall prefer to deviate and set a lower price, to capture the customers that make two-stop shopping.

Since there only exists equilibrium with one-stop shopping when  $n \leq 4$  and with two-stop shopping when  $7 \leq n \leq 11$ , we conclude that there is no equilibrium (in pure strategies) when the number of goods is  $5 \leq n \leq 6$  or  $n \geq 12$ .

### 3.2 Competition between two department stores

Now, we consider the case in which there are two department stores, one at each extreme of the city. Each department store chooses the price to charge for each of the  $n$  products, with the objective of maximizing its profit, taking as given the prices set by the other department store.

**Proposition 4.** *In the case of competition between two department stores:*

(1) *The price of the bundle is equal to the transportation cost parameter:*

$$P_L = P_R = t, \quad \text{with} \quad \sum_{i=1}^n |p_{iL} - p_{iR}| \leq t.$$

(2) *Consumers make all their purchases at the closest department store:*

$$q_{iL} = q_{iR} = \frac{1}{2}.$$

(3) *The resulting profits are also independent of the number of goods:*

$$\Pi_L = \Pi_R = \frac{t}{2}.$$

*Proof.* See the Appendix. □

In equilibrium, the department stores charge the same price for the bundle of  $n$  goods (there is, once more, some indeterminacy regarding the split of the bill between the goods). It is,

then, obvious that no consumer is willing to travel to both extremes of the city. They all buy the  $n$  goods at the department store that is closer.

What may be surprising is that the margin (difference between price and marginal cost) with  $n$  goods is the same as in the standard Hotelling model, in which a single good is sold. The reason why the margin is not greater with  $n$  goods is related to the fact the reservation utility of the customer is not relevant for the pricing decisions of the firms (as long as it is high enough, as is typically assumed). With one-stop shopping, the  $n$  goods are equivalent to a single bundled good. Therefore, even if customers attribute a higher utility to the  $n$  goods than to a single good, the margin remains constant and equal to the transportation cost parameter.<sup>14</sup>

### 3.3 Competition between two shopping malls

In the case of competition between two shopping malls (one at each extreme of the city), the shops that sell the same good at different locations are direct competitors. However, their demand also depends on the prices of the other goods. This interdependence across shops selling different goods exists because, when deciding where to buy each good, consumers take into account not only the price but also the transportation costs that they have to support. A shop benefits from having low prices for the goods sold at the same location (since this attracts customers to its location); and high prices for the goods sold at the other location (since this repels customers from the other location). But since this externality has no influence on the pricing decisions of the shops, the equilibrium of the model replicates that of the single-product model.

**Proposition 5.** *In the case of competition between two shopping malls:*

(1) *The price of each good is equal to the transportation cost parameter:*

$$p_{iL} = p_{iR} = t, \quad \forall i \in \mathcal{I}.$$

(2) *Consumers make all their purchases at the closest shopping mall:*

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<sup>14</sup>The same occurs in the case of Bertrand competition with homogeneous products. Independently of the number of products that firms sell, their equilibrium margin is always null.

$$q_{iL} = q_{iR} = \frac{1}{2}, \forall i \in \mathcal{I}.$$

(3) *The profit of each firm is also independent of the number of goods:*

$$\Pi_{iL} = \Pi_{iR} = \frac{t}{2}, \forall i \in \mathcal{I}.$$

*Proof.* See the Appendix. □

The joint profit of the  $n$  shops located at each shopping mall is greater than the profits obtained in any of the alternative scenarios that we have considered.

## 4 Endogenous modes of retail

Until now, we have assumed that the organization of each shopping center was exogenous. In this section, we analyze whether it is more profitable for a shopping center to be organized as a department store or as a shopping mall.

More precisely, we consider a two-stage game in which the shopping centers start by simultaneously deciding their modes of retail (department store or shopping mall) and then compete in prices. The resulting payoffs are, depending on the chosen mode of retail, the profits of the department store or the joint profits of the shops at a mall, in the equilibrium of the corresponding scenario.<sup>15</sup>

For  $n \leq 4$ , the profits of the shopping centers in each competitive scenario are shown in Table 1. We conclude that shopping malls have higher profits than department stores, regardless of whether they compete against a department store or a shopping mall.

**Proposition 6.** *If  $n \leq 4$ , it is a dominant strategy for a shopping center to organize itself as a shopping mall.*

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<sup>15</sup>We do not consider the possibility of some shops merging and others remaining separated. Opening the possibility of partial mergers can change the outcome of the game, as suggested by the contributions of Salant, Switzer and Reynolds (1983) and Kamien and Zang (1990).

	Department Store	Shopping Mall
Department Store	$(\frac{1}{2}t, \frac{1}{2}t)$	$(\frac{(2n+1)^2}{2(n+2)^2}t, \frac{9n}{2(n+2)^2}t)$
Shopping Mall	$(\frac{9n}{2(n+2)^2}t, \frac{(2n+1)^2}{2(n+2)^2}t)$	$(\frac{n}{2}t, \frac{n}{2}t)$

Table 1: Profits of the competing shopping centers, for  $n \leq 4$ .

For  $7 \leq n \leq 11$ , the payoff matrix is given by Table 2. In this case: to compete against a department store, it is better to be organized as a department store; while to compete against a shopping mall it is better to be organized as a shopping mall.

	Department Store	Shopping Mall
Department Store	$(\frac{1}{2}t, \frac{1}{2}t)$	$(\frac{(n-1)^2}{(n+1)^2}t + \frac{4}{9}t, \frac{4(n-1)}{(n+1)^2}t + \frac{1}{9}t)$
Shopping Mall	$(\frac{4(n-1)}{(n+1)^2}t + \frac{1}{9}t, \frac{(n-1)^2}{(n+1)^2}t + \frac{4}{9}t)$	$(\frac{n}{2}t, \frac{n}{2}t)$

Table 2: Profits of the competing shopping centers, for  $7 \leq n \leq 11$ .

**Proposition 7.** *If  $7 \leq n \leq 11$ , there are two equilibria: competition between two shopping malls and competition between two department stores.*

Of course, the shopping centers would prefer to coordinate on the equilibrium with two shopping malls. Therefore, we should expect to have shopping malls at both extremes of the city.

## 5 Welfare analysis

### 5.1 Social welfare

In this model, the total demand is perfectly inelastic (each consumer buys one unit of each good that is available in the market, independently of the prices of the goods). Therefore, a change in prices only leads to a transfer of surplus between consumers and firms. The total

surplus remains unaltered. In this context, the maximization of total surplus is equivalent to the minimization of the total transportation costs incurred by consumers.

It is clear that the total transportation costs are minimized when each consumer shops at the closest store. This occurs in the case of competition between two department stores and in the case of competition between two shopping malls. When there is a department store competing with a shopping mall, the indifferent consumer is no longer located at the middle of the city. If  $n \leq 4$ , there are more consumers shopping at the department store than at the shopping mall ( $\tilde{x} > \frac{1}{2}$ ). If  $7 \leq n \leq 11$ , there are consumers who shop at both extremes of the city. The total transportation costs are even higher in such a situation. Thus, the existence of different modes of retail diminishes the total surplus.

## 5.2 Consumers' surplus

Let  $CS_{DD}$ ,  $CS_{MM}$  and  $CS_{DM}$  denote the consumers' surplus in each of the three scenarios: (DD) competition between two department stores; (MM) competition between two shopping malls; and (DM) competition between a department store and a shopping mall.

When the mode of retail is the same in both extremes of the city, the price of the bundle of goods is the same at both shopping centers. As a result, the indifferent consumer is located at the middle of the city and the total transportation cost is minimized. We have:

$$CS_{DD} = \tilde{x}(V - t) + \tilde{x}(V - t) - t \int_0^{\tilde{x}} x \, dx - t \int_{\tilde{x}}^1 (1 - x) \, dx = V - \frac{5}{4}t;$$

$$CS_{MM} = \tilde{x}(V - nt) + \tilde{x}(V - nt) - t \int_0^{\tilde{x}} x \, dx - t \int_{\tilde{x}}^1 (1 - x) \, dx = V - \frac{4n + 1}{4}t,$$

where  $V = \sum_{i \in \mathcal{I}} V_i$ .

When a department store competes with a shopping mall and the number of goods is less than or equal to 4 (equilibrium with one-stop shopping), the consumers' surplus is:

$$CS_{DM} = \int_0^{\tilde{x}} (V - P_L - tx) \, dx + \int_{\tilde{x}}^1 [V - P_R - t(1 - x)] \, dx$$

$$= V - \frac{10n^2 + 28n + 7}{4(n + 2)^2}t.$$

When  $7 \leq n \leq 11$  (equilibrium with two-stop shopping), it is given by:

$$\begin{aligned} CS_{DM} &= \int_0^{\tilde{x}_L} (V - P_L - tx) \, dx + \int_{\tilde{x}_L}^{\tilde{x}_R} (V - P_{LR} - t) \, dx + \int_{\tilde{x}_R}^1 [V - P_R - t(1-x)] \, dx \\ &= V - \frac{19n^2 + 20n - 17}{9(n+1)^2} t. \end{aligned}$$

From the point of view of consumers, competition between department stores is the most favorable scenario. Prices are lower than in the other scenarios, and transportation costs are minimized. It is not so straightforward to compare the case of competition between two shopping malls (lower transportation costs) with the case of competition between a shopping mall and a department store (lower prices). We find that the price effect dominates. Consumers prefer competition between a shopping mall and a department store rather than competition between two shopping malls.

**Proposition 8.** *Comparing the consumer's surplus in the three competitive scenarios, for  $2 \leq n \leq 4$  and for  $7 \leq n \leq 11$ , we obtain:*

$$CS_{DD} > CS_{DM} > CS_{MM}.$$

It is somewhat surprising that the lower the number of independent stores in the market, the higher the consumers' surplus. This result contradicts the typical intuition, according to which as the number of firms in the market increases, competition becomes stronger, leading to lower prices. This is not the case, since the price for the bundle of goods is cheaper when there are only two department stores.

## 6 Conclusions

We have developed a multi-product version of the model of Hotelling (1929) to study competition between shopping centers that can be organized as department stores or as shopping malls. In particular, we analyzed how the modes of retail affect prices, market shares and profits, and which retail structures are more likely to emerge endogenously.

Comparing the competitive behavior of a department store with that of a shopping mall, we found (as in previous works) that the department store competes more aggressively. This occurs because a department store, when choosing prices, takes into account that a price drop in one good increases the demand for all its goods. In contrast, a shop at a mall only takes into account its individual demand when choosing the price to charge for its good.

When a department store competes with a shopping mall, the bundle of goods is cheaper at the department store than at the shopping mall. Nevertheless, the demand-effect more than compensates the price-effect and the department store obtains higher profits than the shops at the mall taken together. In spite of having higher profits, the department store has incentives to separate itself into a shopping mall. If the shops at each extreme decide whether to organize themselves as a shopping mall or as a department store, the competitive scenario that is expected to emerge is competition between two shopping malls.<sup>16</sup>

The most distinctive feature of our work, with respect to the existing literature, is that we do not restrict consumers to make all their purchases in a single place. Still, we found that when the mode of retail is the same in the two shopping centers, no consumer finds it worthwhile to visit both places (“one-stop shopping” holds in equilibrium). However, if there is one department store competing with one shopping mall, there may be consumers traveling to both extremes of the city to buy each product where it is cheaper (this occurs if the number of goods for sale is between seven and eleven).

To understand why the “one-stop shopping” assumption may be too strong, suppose that there are nine different goods for sale at a shopping mall, each and every one at a price equal to 1. With the transportation cost being equal to 5, the best response of a department store would be to price the bundle at  $P_L = 7$ . With “one-stop shopping”, the department store would have a demand equal to 0.7 and a profit of 4.9. But the department store can also induce “two-stop shopping”, by charging a price of 3 for one of the goods and a price of 4 for the bundle of the remaining eight goods. The demand for the expensive good would decrease to 0.6, but the demand for the other goods would increase to 0.8. As a result, the profit would increase to 5.

The marketing strategy that we find to be most profitable in the presence of two-stop

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<sup>16</sup>This is actually the competitive scenario that consumers desire the least. Having two shopping malls or two department stores is equally optimal in terms of total surplus. However, consumers are better off in the case of competition between two department stores (since prices are lower).

shopping is to set a high price for a single good and low prices for the remaining goods. Instead of the well-known *loss leader* strategy (a single product sold at a low price to attract customers to the store), we find that it is optimal to follow a pricing strategy that is, in some sense, opposite: to sell a single product at a high price to profit from the customers that have visited the store.

If the number of goods is five, six or higher than eleven, there is no equilibrium in pure strategies for the case of competition between a department store and a shopping mall. Therefore, a possible extension of this work is to allow for mixed strategies in the price competition stage. This could be compatible with having informed consumers. The shops would set prices simultaneously and irreversibly, and then consumers would observe the prices and decide their purchases. An alternative would be to consider uninformed consumers. They would have to travel to a shopping center to observe the prices that were actually set, and then decide whether to travel also to the other shopping center or not. We leave this extension for future work.

## 7 Appendix

### Proof of Lemma 3.

Inspection of the profit function (2) shows that  $\Pi_L$  depends positively on the price of the bundle  $\mathcal{I}_L$  at the right extreme ( $\sum_{i \in \mathcal{I}_L} p_{iR}$ ) with coefficient  $\frac{1}{t} \sum_{i \in \mathcal{I}_L} p_{iL}$ , and on the price of the bundle  $\mathcal{I}_R$  at the right extreme ( $\sum_{i \in \mathcal{I}_R} p_{iR}$ ) with coefficient  $\frac{1}{t} \sum_{i \in \mathcal{I}_R} p_{iL}$ .

From condition (6), the latter coefficient is lower than the former whenever two-stop shopping is preferred.

Observe that the choice of  $\mathcal{I}_L$  and  $\mathcal{I}_R$  only impacts the attainable profit through the distribution of  $P_R$  between  $\sum_{i \in \mathcal{I}_L} p_{iR}$  and  $\sum_{i \in \mathcal{I}_R} p_{iR}$ .

Thus, it is in the interest of the department store to choose the bundle  $\mathcal{I}_R$  that minimizes  $\sum_{i \in \mathcal{I}_R} p_{iR}$  (and the bundle  $\mathcal{I}_L$  that maximizes  $\sum_{i \in \mathcal{I}_L} p_{iR}$ ). This is achieved when  $\mathcal{I}_R$  only contains the good that is the cheapest at  $R$ .

The set  $\mathcal{I}_R$  can only contain more goods if these have null price at  $R$ .  $\square$

### Proof of Proposition 2.

To finish the proof, we must verify that  $p_{iR} = \frac{3}{n+2}t$  maximizes the profit of the shop at the mall that sells good  $iR$ . So far, we only know that it is a maximum in  $D_3$ , as long as  $\sum_{i=1}^n |p_{iL} - \frac{3}{n+2}t| \leq t$  (which implies that it is inside the domain).

With  $P_L = \frac{2n+1}{n+2}t$  and  $p_{jR} = \frac{3}{n+2}t, \forall j \neq i$ , the profit function has a local maximum in  $D_3$ , attained at  $p_{iR}^* = \frac{3}{n+2}t$ . The derivative of the demand with respect to price is  $-\frac{1}{t}$  in  $D_3$  and  $-\frac{1}{t}$  in  $D_4$ . Therefore, if a marginal price increase is not profitable, it is also not profitable to make a greater price increase to  $D_4$ , where the demand is more price-sensitive. The maximum is either  $p_{iR}^* = \frac{3}{n+2}t$  or attained in  $D_2$ .

In  $D_2$ , the demand for good  $iR$  is  $q_{iR} = \frac{1}{t}(s_{Ri} + p_{iL} - p_{iR})$  and the corresponding profit is  $\Pi_{iR} = \frac{1}{t}(s_{Ri}p_{iR} + p_{iL}p_{iR} - p_{iR}^2)$ . The first-order condition is satisfied at  $p_{iR}^{**} = \frac{s_{Ri}}{2} + \frac{p_{iL}}{2}$ , implying that:

$$\Pi_{iR}(p_{iR}^{**}) = \frac{(s_{Ri} + p_{iL})^2}{4t}.$$

We are sure that  $p_{iR}^* = \frac{3}{n+2}t$  is a global maximizer if:

$$\Pi_{iR}(p_{iR}^{**}) \leq \Pi_{iR}(p_{iR}^*) \Leftrightarrow \frac{(s_{Ri} + p_{iL})^2}{4t} \leq \frac{9}{2(n+2)^2}t \Leftrightarrow s_{Ri} + p_{iL} \leq \frac{\sqrt{18}}{n+2}t.$$

Otherwise, the alternative maximizer,  $p_{iR}^{**} = \frac{s_{Ri}}{2} + \frac{p_{iL}}{2}$ , upsets our equilibrium if it belongs to the domain,  $D_2$ . This occurs if:

$$\begin{cases} \frac{s_{Ri}}{2} + \frac{p_{iL}}{2} \leq -t + p_{iL} + s_{Li} + s_{Ri} \\ \frac{s_{Ri}}{2} + \frac{p_{iL}}{2} \geq -t + p_{iL} + s_{Ri} \end{cases}. \quad (13)$$

In (the candidate) equilibrium, we have  $P_R - P_L = \frac{n-1}{n+2}t$ . As a result:

$$\sum_{j \in I \setminus \{i\}} (p_{jR} - p_{jL}) + \frac{3}{n+2}t - p_{iL} = \frac{n-1}{n+2}t \Leftrightarrow s_{Li} = \frac{n-4}{n+2}t + s_{Ri} + p_{iL}. \quad (14)$$

Substituting (14) in (13), we find that  $p_{iR}^{**}$  is in the domain  $D_2$  when:

$$s_{Ri} + p_{iL} \in \left[ \frac{4}{n+2}t, 2t \right].$$

We surely have  $s_{Ri} + p_{iL} \leq 2t$ , because:

$$s_{Li} \leq t \Leftrightarrow \frac{n-4}{n+2}t + s_{Ri} + p_{iL} \leq t \Leftrightarrow s_{Ri} + p_{iL} \leq \frac{6}{n+2}t.$$

Then, if  $s_{Ri} + p_{iL} > \frac{\sqrt{18}}{n+2}t$ , we have  $p_{iR}^{**}$  in  $D_2$  and it upsets our candidate equilibrium,  $p_{iR}^*$ . As a result, our candidate is an equilibrium if and only if,  $\forall i \in \mathcal{I}$ :

$$s_{Ri} + p_{iL} \leq \frac{\sqrt{18}}{n+2}t \Leftrightarrow s_{Li} \leq \frac{n + \sqrt{18} - 4}{n+2}t.$$

If  $\mathcal{I}_R$  is non-empty, the conditions that bind are those for  $i \in \mathcal{I}_R$  (because  $s_{Li}$  is maximal), which can be written as:

$$\sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| \leq \frac{n + \sqrt{18} - 4}{n+2}t.$$

If  $\mathcal{I}_R$  is empty, then all the above conditions are surely satisfied because, in this case:

$$\sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| = \sum_{i \in \mathcal{I}} (p_{iR} - p_{iL}) = P_R - P_L = \frac{n-1}{n+2}t < \frac{n + \sqrt{18} - 4}{n+2}t.$$

To obtain a more elegant condition, notice that (in the candidate equilibrium):

$$\begin{aligned} P_R - P_{LR} - P_L + P_{LR} &= \frac{n-1}{n+2}t \Leftrightarrow \sum_{i \in \mathcal{I}_R} |p_{iL} - p_{iR}| = \sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| - \frac{n-1}{n+2}t \Leftrightarrow \\ \Leftrightarrow \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| &= 2 \sum_{i \in \mathcal{I}_L} |p_{iR} - p_{iL}| - \frac{n-1}{n+2}t. \end{aligned}$$

Therefore, the equilibrium condition can be written as:

$$\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq \frac{2(n + \sqrt{18} - 4)}{n+2}t - \frac{n-1}{n+2}t = \frac{n + 6\sqrt{2} - 7}{n+2}t.$$

□

### Proof of Proposition 3.

To finish the proof, we need to check that the shops at the mall that are selling goods at a higher price than the department store do not wish to deviate as long as  $7 \leq n \leq 11$ . The candidate equilibrium maximizes their profits in the domain  $D_4$ , but we must guarantee that the shops do not prefer to choose prices in  $D_3$  (if it exists) or in  $D_2$ .

It is convenient to start with some preliminary calculations. In the candidate equilibrium, for the shop that sells good  $i \in \mathcal{I}_L$ , we have:

$$\begin{aligned} s_{Li} &= \sum_{j \in \mathcal{I}_L \setminus \{i\}} (p_{jR} - p_{jL}) = \frac{n-3}{n+1}t + p_{iL}; \\ s_{Ri} &= \sum_{j \in \mathcal{I}_R \setminus \{i\}} (p_{jL} - p_{jR}) = \frac{t}{3}. \end{aligned}$$

We know that  $D_3$  is empty if and only if:

$$s_{Li} + s_{Ri} > t \Leftrightarrow \frac{n-3}{n+1}t + p_{iL} + \frac{t}{3} > t \Leftrightarrow p_{iL} > \frac{11-n}{3(n+1)}t.$$

For  $n \geq 12$ , the domain  $D_3$  is surely empty (for  $n = 11$  it is either empty or a singleton).

Let us start by considering the case in which  $D_3$  is empty and study whether a deviation to  $D_2$  is profitable. From (8), the interior maximizer in  $D_2$  is:

$$p_{iR}^{**} = \frac{p_{iL}}{2} + \frac{t}{6}. \quad (15)$$

This alternative only belongs to  $D_2$  if  $p_{iR}^{**} < p_{iL}$ . Equivalently, if:

$$\frac{p_{iL}}{2} + \frac{t}{6} < p_{iL} \Leftrightarrow p_{iL} > \frac{t}{3}. \quad (16)$$

But since  $p_{iL} < \frac{2}{n+1}t$ , condition (16) cannot hold for  $n \geq 7$ . This means that the optimal choice in  $D_2$  is at the frontier:  $p_{iR} = p_{iL}$  (infinitesimally close, but lower than  $p_{iL}$ ). The

resulting profit is  $\Pi_{iR} = \frac{p_{iL}}{3}$ . We conclude that the shop gains with this deviation if:

$$\frac{p_{iL}}{3} > \frac{4}{(n+1)^2}t \Leftrightarrow p_{iL} > \frac{12}{(n+1)^2}t.$$

It is straightforward to confirm that  $p_{iL} > \frac{12}{(n+1)^2}t$  implies that  $p_{iL} > \frac{11-n}{3(n+1)}t$ , which was a prerequisite (for  $D_3$  to be empty).

Notice that if  $p_{iL} \leq \frac{12}{(n+1)^2}t, \forall i \in \mathcal{I}_L$ , then:

$$\sum_{i \in \mathcal{I}_L} p_{iL} \leq \frac{12(n-1)}{(n+1)^2}t \Leftrightarrow \frac{n-1}{n+1}t \leq \frac{12(n-1)}{(n+1)^2}t \Leftrightarrow n \leq 11.$$

For  $n \geq 12$ , the candidate is not an equilibrium because at least one of the shops deviates.

Now suppose that  $D_3$  is not empty. We must have  $p_{iL} \leq \frac{11-n}{3(n+1)}t$  and  $p_{iL} < \frac{2}{n+1}t$ . The first condition can only hold for  $n \leq 11$ , while the second is implied by the first. In this case, we must consider deviations to  $D_3$  and also to  $D_2$ .

The interior maximum in  $D_2$ , given by (15), is again outside the domain because:

$$p_{iL} \leq \frac{11-n}{3(n+1)}t \Rightarrow \frac{p_{iL}}{2} + \frac{t}{6} > -t + p_{iL} + s_{Li} + s_{Ri}.$$

The candidate deviation is, therefore, at the frontier. But if it is profitable to increase the price until the frontier of  $D_2$  it is surely profitable to keep increasing the price after entering  $D_3$  because the price-sensitivity of demand is lower in  $D_3$ . It is enough to consider deviations in  $D_3$ .

From (9), the candidate deviation is:

$$p_{iR} = \frac{P_L}{2} - \frac{1}{2} \sum_{j \in \mathcal{I} \setminus \{i\}} p_{jR} + \frac{t}{2} = \frac{n+13}{6(n+1)}t.$$

This deviation is outside  $D_3$ , because:

$$\frac{n+13}{6(n+1)}t \leq t + p_{iL} - s_{Li} - s_{Ri} \Leftrightarrow \frac{n+13}{6(n+1)}t \leq \frac{11-n}{3(n+1)}t \Leftrightarrow n \leq 3,$$

which is false.

Therefore, the maximum in  $D_3$  is at the frontier with  $D_4$ . Global continuity of the profit function (satisfied as long as  $D_3$  is not empty) and concavity in  $D_4$  implies that the local maximum at  $D_4$  is actually the global maximum.  $\square$

#### Proof of Proposition 4.

We start by showing that an equilibrium with two-stop shopping is not possible.

With  $n = 2$ , one of the goods must be cheaper at  $x = 0$  and the other must be cheaper at  $x = 1$ . Combining the first-order conditions, (3) and (4), we obtain, for  $i \in \mathcal{I}_L$ :

$$\begin{cases} p_{iL} = \frac{p_{iR}}{2} \\ p_{iR} = \frac{p_{iL}}{2} + \frac{t}{2} \end{cases} \Leftrightarrow \begin{cases} p_{iL} = \frac{1}{3}t \\ p_{iR} = \frac{2}{3}t. \end{cases}$$

For  $j \in \mathcal{I}_R$ , we obtain  $p_{jL} = \frac{2}{3}t$  and  $p_{jR} = \frac{1}{3}t$ . This is an equilibrium, but no consumer gains by shopping at both extremes (only the consumer in the middle,  $x = \frac{1}{2}$ , is indifferent between one-stop shopping and two-stop shopping).

For  $n > 2$ , since both department stores wish to have a single good that is more expensive than at the other extreme, the only possibility is that prices are symmetric for all the goods except two. Being symmetric, they must be null, otherwise the department stores would gain by undercutting each other. Lemma 3 confirms that there can only exist two goods with strictly positive prices (one at each extreme). But this means that the department store at  $x = 0$  is offering the goods in  $\mathcal{I}_L$  for free and, therefore, would prefer to deviate and induce one-stop shopping.

With one-stop shopping, the first-order conditions for the profit-maximization problems of the department stores imply that:

$$P_L = \frac{P_R}{2} + \frac{t}{2} \quad \text{and} \quad P_R = \frac{P_L}{2} + \frac{t}{2},$$

yielding:

$$P_L = P_R = t.$$

It is straightforward to verify that they do not have incentives to deviate (5).

The equilibrium demand and profits follow immediately.  $\square$

**Lemma 4.** *When there are two shopping malls in the city, no consumer shops at both extremes of the city (in equilibrium).*

**Proof of Lemma 4.**

By way of contradiction, suppose that the vector of prices that maximize the profits of the shops is such that  $\mathbf{p} \in \mathcal{P}_2$ . More precisely, that  $0 < \tilde{x}_L < \tilde{x} < \tilde{x}_R < 1$ .

(i) There cannot be any  $i \in \mathcal{I}$  for which  $p_{iL} = p_{iR} > 0$ . If that was the case, the shop selling good  $iL$  could infinitesimally reduce its price and conquer all consumers at  $x \in [\tilde{x}_L, \tilde{x}_R]$ . The shop selling good  $iR$  would have the same incentive to decrease its price.

It cannot also be the case that  $p_{iL} = p_{iR} = 0$  for some  $i \in \mathcal{I}$ . In such a situation, both shops would obtain a null profit. However, the shop selling good  $i$  at  $x = 0$ , could choose  $p_{iL} > 0$  and profit  $\Pi_{iL} = p_{iL}\tilde{x}_L > 0$ . The same argument applies to the shop selling good  $iR$ .

(ii) Since  $p_{iL} \neq p_{iR}$ ,  $\forall i \in \mathcal{I}$ , we have  $\mathcal{I}_L \cup \mathcal{I}_R = \mathcal{I}$ . Thus, if the cardinality of  $\mathcal{I}_L$  is  $k$ , the cardinality of  $\mathcal{I}_R$  is  $n - k$ .

The profit function of the shop that sells the good  $iL$  is:

$$\Pi_{iL} = \begin{cases} p_{iL} \tilde{x}_R, & i \in \mathcal{I}_L \\ p_{iL} \tilde{x}_L, & i \in \mathcal{I}_R \end{cases},$$

while the profit function of the shop selling the good  $iR$  is:

$$\Pi_{iR} = \begin{cases} p_{iR} (1 - \tilde{x}_R), & i \in \mathcal{I}_L \\ p_{iR} (1 - \tilde{x}_L), & i \in \mathcal{I}_R \end{cases}.$$

If  $i \in \mathcal{I}_L$ , the first-order conditions are:

$$\begin{cases} \frac{\partial \Pi_{iL}}{\partial p_{iL}} = 0 \\ \frac{\partial \Pi_{iR}}{\partial p_{iR}} = 0 \end{cases} \Leftrightarrow \begin{cases} p_{iL} = P_R - P_{LR} \\ p_{iR} = t - P_R + P_{LR} \end{cases} \Rightarrow p_{iR} = t - p_{iL}.$$

The expressions above imply that  $\forall i, j \in \mathcal{I}_L$ :  $p_{iL} = p_{jL}$  and  $p_{iR} = p_{jR}$ . Moreover:

$$p_{iL} = \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}) = k(t - 2p_{iL}) \Leftrightarrow p_{iL} = \frac{k}{2k+1}t$$

and

$$p_{iR} = \frac{k+1}{2k+1}t.$$

Analogously, if  $i \in \mathcal{I}_R$ , then:

$$p_{iL} = \frac{n-k+1}{2n-2k+1}t \quad \text{and} \quad p_{iR} = \frac{n-k}{2n-2k+1}t.$$

The expressions for the marginal consumers,  $\tilde{x}_L$  and  $\tilde{x}_R$ , follow immediately:

$$\tilde{x}_L = \frac{n-k+1}{2n-2k+1} \quad \text{and} \quad \tilde{x}_R = \frac{k}{2k+1}.$$

It is straightforward to see that  $\tilde{x}_L > \tilde{x}_R$ . Contradiction.

□

### Proof of Proposition 5.

By Lemma 4 (in this Appendix), there is no equilibrium with prices in  $\mathcal{P}_2$ . Therefore, we must seek prices satisfying the condition  $\sum_{i=1}^n |p_{iL} - p_{iR}| \leq t$ .

As obtained in (11), the first-order conditions of the  $n$  shops at the malls imply that:

$$P_L = \frac{n}{n+1}(t + P_R) \quad \text{and} \quad P_R = \frac{n}{n+1}(t + P_L).$$

Therefore:

$$P_L = P_R = nt.$$

Using (9), we obtain the individual prices:

$$p_{iL} = p_{iR} = t, \forall i \in \mathcal{I}.$$

To complete the proof, we must verify that these local maxima are global maxima. We

need to check if each shop chooses the price  $t$ , when the remainders charge  $t$  for their products. Without loss of generality, we consider the shop that sells good  $iR$ .

Substituting  $p_{jL} = p_{jR} = t, \forall j \neq i$  and  $p_{iL} = t$  in the demand for good  $iR$ , given in (7), we obtain:

$$q_{iR} = \begin{cases} 1 - \frac{p_{iR}}{2t}, & p_{iR} \in [0, 2t] \\ 0, & p_{iR} \in ]2t, +\infty[ \end{cases} .$$

and

$$\Pi_{iR} = \begin{cases} p_{iR} \left(1 - \frac{p_{iR}}{2t}\right), & p_{iR} \in [0, 2t] \\ 0, & p_{iR} \in ]2t, +\infty[ \end{cases} ,$$

The profit function is globally concave and continuous. Therefore, the local maximum is also the global maximum.

The equilibrium demand and profits follow immediately. □

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