Using Cost Observation to Regulate Bureaucratic Firms*

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Abstract. We study regulation of a bureaucratic provider of a public good in the presence of moral hazard and adverse selection. By bureaucratic we mean that it values output in itself, and not only profit. Three different financing systems are studied: cost reimbursement, prospective payment, and the optimal contract. In all cases, the output level increases with the bureaucratic bias. We find that the optimal contract is linear in cost (fixed payment plus partial cost-reimbursement). A stronger preference for high output reduces the tendency of the firm to announce a high cost (adverse selection), allowing a more powered incentive scheme (a lower fraction of the costs is reimbursed), which alleviates the problem of moral hazard.

Keywords: Procurement, Regulation, Adverse selection, Moral hazard, Bureaucracy.

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1 Introduction

Regulation of a firm under asymmetric cost information has been the subject of intensive research since the pioneering papers of Baron and Myerson (1982), Baron and Besanko (1984) and Laffont and Tirole (1986). In this literature, it is assumed that the managers of the firms maximize profit net of the disutility of effort. This is a restrictive assumption, since managers are known to be interested not only in monetary rewards but also in managing a large firm (Williamson, 1974). Such preference may reflect the concern of the managers with their reputation and career.

In fact, large firms tend to develop bureaucratic management structures which lead to systematic deviations from profit-maximizing behavior (Monsen and Downs, 1965).[1] One of these deviations is a bias toward revenue maximization. This was found by Amihud and Kamin (1979), who gave empirical support to Baumol's (1958, 1959) hypothesis of revenue maximization as the objective of large firms (see also Maris (1963) and the references therein). A reason behind this bias may be the observed relation between past revenue and current managerial compensation (McGuire, Chiu and Elbing, 1962).

These issues seem to be even more important in public firms, which are characterized by weak external control on efficiency and weak internal incentives (Mueller, 2003). To describe the behavior of bureaucrats in the public administration, Niskanen (1971) put forward the hypothesis of budget maximization. He remarked that the goals of the bureaucrat are "*salary, perquisites of the office, public reputation, power, patronage, output of the bureau, ease of making changes, and ease in managing the bureau*" (Niskanen, 1971).[2]

The possibility of bureaucratic behavior should thus be taken into account when designing a financing system. In this paper, we study procurement contracts between the government and a bureaucratic firm, and examine whether a preference for higher output changes the results previously obtained in the literature.

In the theory of regulation and procurement, it is usually assumed that the firm (agent) is better informed about its cost function than the regulator (principal). This is common to the contributions of Baron and Myerson (1982), Baron and Besanko (1984) and Laffont and Tirole (1986, 1993), which were important milestones.

[1]Recent empirical evidence of deviations from profit-maximization was provided by Chetty and Saez (2005) and Brown, Liang and Weisbenner (2008), who studied the response of corporations to the 2003 dividend tax cut in the USA.

[2]For a discussion on the motivation of bureaucrats in a public organization, see Wilson (1989, ch. 9).

Baron and Myerson (1982) studied the case in which the realized cost is unobservable. The gross payment to the firm could only be a function of the cost function announced by the firm (prospective payment). In this context, the firm tended to announce a high marginal cost, in order to receive a high payment while incurring in a low cost. The procurement contract should provide incentives for the firm to announce its true marginal cost (rewarding the firm for announcing a low marginal cost). In the setup of Baron and Besanko (1984), the regulator could, *ex post*, pay an auditing cost to observe (imperfectly) the firm's realized cost. The optimal scheme is to audit the firm when the reported cost is above a particular level and impose a penalty when the observed cost is low.

To this context of regulation under adverse selection, Laffont and Tirole (1986) add the problem of moral hazard. While in the models of Baron and Myerson (1982) and Baron and Besanko (1984) the firm's single decision variable is the announcement of its unobservable marginal cost (adverse selection), in the model of Laffont and Tirole (1986), the firm's unobservable level of cost-reducing effort is also a decision variable (moral hazard). In this case, the planner cannot penalize low observed costs because the firm would simply reduce its effort to increase cost. The optimal contract is shown to be linear in observed cost, being composed by a fixed payment plus a partial cost reimbursement. Firms with lower marginal cost produce higher output, and make more effort. To induce this greater effort on the part of more efficient firms, the fraction of realized cost that the government reimburses to the firm is decreasing with the firm's efficiency. The net payment that the firm receives is also increasing with efficiency, more than compensating for the higher disutility of effort that is demanded. This is necessary for the firm to announce its efficiency truthfully.[3]

Technically, we extend the model of Laffont and Tirole (1986) to allow the manager of the firm to have a preference for higher output, deriving utility from the difference between the output level of the firm and a reference output level. The social welfare function that the government seeks to maximize may incorporate, or not, this component of the firm's objective function (we consider both cases).[4] In this setup, we study three different kinds of financing systems: a cost reimbursement system, which consists of compensating the firm for the costs in which it incurs, a prospective payment system, which grants a fixed

[3]Laffont and Tirole (1986) compare their setting with the case in which the regulator is unable to observe cost (as in Baron and Myerson, 1982). If the cost is unobservable, the optimal regulatory policy is a gross transfer that depends on the firm's announced marginal cost (prospective payment) in such a way that the firm has no incentive to misrepresent its costs. The prospective payment implies no effort distortion for a given output level, contrary to the optimal incentive contract with cost observability, in which effort is lower than optimal.

[4]Incorporating the bureaucratic bias in the government's welfare function results in higher output and effort. Naturally, the firm's utility increases, while consumers' welfare decreases.

financing, independently of the costs that the firm comes to incur, and the optimal incentive scheme, which is found to remain linear in observed cost (in spite of the preference for higher output).

In all cases, the output level is shown to be increasing with the strength of the preference for higher output. Since the cost savings associated with the effort are proportional to the output, the effort level is increasing with the output level, and, therefore, with the bureaucratic bias (except in the context of the cost reimbursement system, in which cost-reducing efforts are not available). We show that the optimal contract remains linear in cost, but depends on the strength of the preference of the firm for higher output. A stronger preference for high output reduces the tendency of the firm to announce a high cost (adverse selection), allowing a more powered incentive scheme (a lower fraction of the costs is reimbursed), which, in turn, alleviates the problem of moral hazard. Independently of the financing system, the expected social welfare increases (decreases) with the bureaucratic bias whenever the expected output is larger (lower) than the reference output level.

The paper is organized as follows. Section 2 describes the model and section 3 analyzes the benchmark case of complete information. In sections 4, 5 and 6, we derive the different procurement contracts: optimal incentive scheme, cost reimbursement and prospective payment, respectively. Finally, Section 7 offers some concluding remarks.

2 The model

We consider a model of procurement in which the government (principal) offers a contract to a firm (agent) for the provision of a public good. It is an extension of the model of Laffont and Tirole (1986) which allows for bureaucratic behavior. The technical analysis follows closely that of Laffont and Tirole (1986).

The firm produces an observable level of output, $q \in [0, \overline{q}]$, incurring in a constant marginal cost, $\hat{\beta} - e$, which depends on the intrinsic marginal cost of the firm, $\hat{\beta} \in \left[\underline{\beta}, \overline{\beta}\right]$, and on a cost-reducing effort level, $e \geq 0$. The intrinsic marginal cost, drawn from a uniform distribution, is firm's private information (adverse selection). The level of effort, chosen by the firm after the contract is signed, is also unobservable by the government (moral hazard).

The total cost is observable, and given by:

$$C = (\hat{\beta} - e)q + \epsilon,$$

where ϵ is a random variable with zero mean describing an *ex post* cost disturbance.

We assume that the firm, instead of maximizing profit net of the disutility of effort (Laffont and Tirole, 1986), has also a preference for higher output. An output that is higher (lower) than a reference level, q_{ref}, yields satisfaction (dissatisfaction).

The *ex ante* utility level of the firm is:

$$U = E_\epsilon(t) + \delta(q - q_{ref}) - \psi(e), \tag{1}$$

where $E_\epsilon(t)$ is the expected value (with respect to ϵ) of the net payment received from the government (the gross payment is $t + C$), $\delta \geq 0$ is the marginal utility of output, q_{ref} is the output reference level, and $\psi(e)$, stands for the disutility of effort, with $\psi'(e) > 0$, $\psi''(e) > 0$ and $\psi'''(e) \geq 0$.

The firm accepts the contract if and only if it leads to $U \geq 0$.[5]

The social value of the public good is $S(q)$, with marginal social value strictly positive and decreasing, $S'(q) > 0$ and $S''(q) < 0$, for any $q \in [0, \overline{q})$. We also set $S(0) = 0$ and $S'(\overline{q}) = 0$ (thus $\overline{q}$ fully covers the needs of the population).

Public good provision is financed by a distortionary mechanism (taxes, for example) so that the social cost of raising one unit is $1 + \lambda$. The welfare of consumers is the social value of the public good net of the cost of providing it:

$$S(q) - (1 + \lambda)E_\epsilon(t + C).$$

For the objective of the government, W, we consider two possibilities:

(i) the government maximizes the sum of the consumer's welfare with the utility of the firm, (this case corresponds to setting $k = 0$, below);

(ii) the government maximizes the sum of the consumer's welfare with the profit of the firm, net of the disutility of effort (corresponds to setting $k = 1$, below).

In fact, we also allow for intermediate cases (any $k \in [0, 1]$, below).

The problem of the government is:

$$\max_{q,e,t} \int_{\underline{\beta}}^{\overline{\beta}} S(q) - (1 + \lambda)E_\epsilon(t + C) + U - k\delta(q - q_{ref}) \; d\hat{\beta}$$

subject to

$$U \geq 0.$$

[5] Observe that the only effect of the output reference level is to shift the participation constraint. It would be equivalent to consider $U = E_\epsilon(t) + \delta q - \psi(e)$, with participation for $U \geq \delta q_{ref}$.

To wrap up, we make the timing of the game explicit:

1. The firm observes its intrinsic marginal cost, $\hat{\beta}$ (adverse selection);

2. The government proposes a contract which specifies an output, $q(\beta)$, and a net payment, $t(\beta, C)$, that depend on the intrinsic marginal cost that is announced by the firm, β, and on the observed total cost, C;

3. The firm accepts (or rejects) the contract, announces β, and chooses an unobservable level of effort, e (moral hazard);

4. The government observes the output, q, and the total cost, C, and makes the corresponding payment, $t(\beta, C) + C$, to the firm.

We adapt Assumption 1 in Laffont and Tirole (1986), which guarantees that: (i) the participation constraint is binding; (ii) the second order conditions are satisfied; (iii) the optimal output is positive; and (iv) the marginal cost is positive.

Assumption 1.

(i) $\lambda > 0$;
(ii) $\forall q \in [0, \overline{q}],\ S''(q)\psi''(0) < -(1+\lambda)$;
(iii) $S'(0) > (1+\lambda)(\overline{\beta} - \delta) + k\delta$;
(iv) $\psi'(\underline{\beta} - \delta) > \overline{q}$.

3 The case of complete information

As a benchmark case, we start by considering that the government is able to observe the intrinsic marginal cost, $\hat{\beta}$, as well as the level of effort, e.

The problem of the government is to maximize social welfare:

$$\max_{q,e,t} \{S(q) - (1+\lambda)E_{\epsilon}(t+C) + U - k\delta(q - q_{ref})\} \tag{2}$$

subject to

$$U \geq 0.$$

We find that the optimal level of output, q_c^*, and the optimal level of effort, e_c^*, are decreasing functions of the intrinsic marginal cost, $\hat{\beta}$, and increasing functions of the bureaucratic bias toward higher output, δ (see Appendix A.1).

We also find that social welfare, W_c^*, is increasing (decreasing) with the bureaucratic bias, δ, if and only if the optimal output, q_c^*, is higher (lower) than the reference output, q_{ref}.

It is not surprising that a firm with a stronger preference for output produces more (effort also increases because the resulting cost savings are proportional to output). This effect is stronger if the government incorporates the preference for output in the social welfare function ($k = 0$). But it is still present even if the government neglects the welfare effect of such bureaucratic satisfaction ($k = 1$), because a firm with a stronger preference for output is willing to receive a lower money transfer, and the government prefers to "pay" with output than to pay with money (to avoid the tax distortion). Notice that the welfare effect of the bureaucratic bias disappears when $k = 1$ and $\lambda \to 0$.

The welfare effect of the bureaucratic bias depends on whether the firm considers that output is high (the glass is half full), implying an increase in utility, or small (the glass is half empty), implying a utility penalty. Again, even if the social welfare function does not include this bureaucratic satisfaction or dissatisfaction, the resulting variation of the money transfer that the firm requires to participate induces an effect in the same direction but with a smaller magnitude (associated with the tax distortion). When the reference output is lower than expected output, a manager who is more "bureaucratic" is less costly to society as a whole, because the manager receives in a non-monetary form a larger part of the informational rent. The opposite occurs when the reference output is higher than expected output.

4 The optimal incentive scheme

In this section, we consider the case in which the government is not able to observe the intrinsic marginal cost of the firm, $\hat{\beta}$, nor the effort level chosen by the firm, e.[6]

The government offers a contract to the firm, $[q(\beta), e(\beta), t(\beta, C)]$, specifying an output, $q(\beta)$, an effort level, $e(\beta)$, and a payment scheme, $t(\beta, C)$, which depend on the intrinsic marginal cost announced by the firm, β.

A firm with an intrinsic marginal cost $\hat{\beta}$ that announces an intrinsic marginal cost β will choose an individually optimal level of effort, attaining an utility given by:

$$U(\beta, \hat{\beta}) = \max_e \left\{ E_\epsilon \left\{ t[\beta, (\hat{\beta} - e)q + \epsilon] \right\} + \delta \left[q(\beta) - q_{ref} \right] - \psi(e) \right\}.$$

[6]Before the contract is signed, the government knows the objective function of the firm, and the prior probability distribution of the efficiency parameter, $\hat{\beta}$.

Thanks to the Revelation Principle, we restrict the contract to be incentive-compatible.[7] It must induce the firm to choose the specified effort, $e = e(\beta)$, and reveal truthfully its intrinsic marginal cost, $\beta = \hat{\beta}$:

$$e(\beta) \in \operatorname*{argmax}_{e} \left\{ E_\epsilon \left\{ t[\beta, (\hat{\beta} - e)q + \epsilon] \right\} + \delta \left[q(\beta) - q_{ref} \right] - \psi(e) \right\}$$

and

$$\hat{\beta} \in \operatorname*{argmax}_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} U(\beta, \hat{\beta}). \tag{3}$$

Incentive compatibility is satisfied if and only if $V'(\hat{\beta}) = -\psi' \left[e(\hat{\beta}) \right]$ and $e'(\hat{\beta}) < 1$, where $V(\hat{\beta})$ is the utility that is attainable by the firm (see Appendix A.2.1). This means that more efficient firms obtain higher utility and that the actual marginal cost, $\hat{\beta} - e$, increases with the intrinsic marginal cost, $\hat{\beta}$.

The problem of the government (solved in Appendix A.2.2) can be written as:

$$\max_{q(\hat{\beta}), e(\hat{\beta}), V(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta}) \right] - (1+\lambda) \left\{ V(\hat{\beta}) - \delta \left[q(\hat{\beta}) - q_{ref} \right] + \psi \left[e(\hat{\beta}) \right] \right\} -$$
$$-(1+\lambda) \left[\hat{\beta} - e(\hat{\beta}) \right] q(\hat{\beta}) + V(\hat{\beta}) - k\delta \left[q(\hat{\beta}) - q_{ref} \right] \, d\hat{\beta} \tag{4}$$

subject to, for all $\hat{\beta}$,

$$V(\hat{\beta}) = 0, \tag{5}$$

$$V'(\hat{\beta}) = -\psi' \left[e(\hat{\beta}) \right], \tag{6}$$

$$e'(\hat{\beta}) < 1. \tag{7}$$

Let $\left\{ q^*(\hat{\beta}), e^*(\hat{\beta}), V^*(\hat{\beta}) \right\}$ denote the solution, let $C^*(\hat{\beta}) = \left[\hat{\beta} - e^*(\hat{\beta}) \right] q^*(\hat{\beta})$, and let $t^*(\hat{\beta}) = V^*(\hat{\beta}) - \delta \left[q^*(\hat{\beta}) - q_{ref} \right] - \psi \left[e^*(\hat{\beta}) \right]$.

When there is no cost disturbance, to implement the solution, it suffices for the government to: (i) ask the firm to announce its marginal cost, β; (ii) choose output $q^*(\beta)$; and (iii) give transfer $t^*(\beta)$ if $C = C^*(\beta)$, and $-\infty$ otherwise.

However, this "knife-edge" mechanism does not work if there is any cost disturbance because the probability of incurring in an extreme penalty becomes positive and makes the firm

[7] By the Revelation Principle (Myerson, 1979), given a Bayesian equilibrium of a game with incomplete information, there exists an incentive compatible direct mechanism that has an equivalent equilibrium.

unwilling to participate. To implement this optimal solution in the more general case of cost disturbance, we must find a transfer function $t(\beta, C)$ that is such that:

$$\left[\hat{\beta}, e^*(\hat{\beta})\right] \in \underset{\beta,e}{\operatorname{argmax}} \left\{ E_\epsilon \left\{ t\left[\beta, (\hat{\beta}-e)q(\beta)+\epsilon\right]\right\} + \delta\left[q(\beta)-q_{ref}\right] - \psi(e)\right\}$$

and

$$E_\epsilon \left\{ t\left\{\hat{\beta}, \left[\hat{\beta}-e^*(\hat{\beta})\right] q^*(\hat{\beta})+\epsilon\right\}\right\} = t^*(\hat{\beta}).$$

These conditions are satisfied by the following transfer function, linear in observed cost:

$$t(\beta, C) = t^*(\beta) + K^*(\beta)\left[C^*(\beta) - C\right], \quad \text{with} \quad K^*(\beta) = \frac{\psi'\left[e^*(\beta)\right]}{q^*(\beta)}. \tag{8}$$

If the observed cost is higher (lower) than expected, $C > C^*(\beta)$, the government reimburses (receives) a fraction of the difference, $1 - K^*(\beta)$, while the firm supports the remaining fraction, $K^*(\beta)$. The payment scheme is, then, composed by a fixed payment, $C^*(\hat{\beta})$, plus a partial cost reimbursement, $[1 - K^*(\beta)]\left[C^*(\beta) - C\right]$.

Proposition 1.

Under Assumption 1, the optimal incentive compatible allocation, $[q^*(\hat{\beta}), e^*(\hat{\beta}), t^*(\hat{\beta})]$, can be implemented by a payment scheme that is linear in observed cost, $t(\beta, C)$.

Proof. See Supp. Material C.5 or follow the steps in Laffont and Tirole (1986). □

As in Laffont and Tirole (1986), the linear scheme implements the optimal allocation and it is the only contract that implements the optimal allocation for any probability distribution of the cost disturbance (see Supplementary Material C.6).

We find that, as in the case of complete information, the optimal level of output, q^*, and the optimal level of effort, e^*, are decreasing functions of the intrinsic marginal cost, $\hat{\beta}$. The more efficient is the firm, the higher are the output and the effort (see Appendix A.2.2).

We also find that the optimal levels of effort and output are increasing with the manager's marginal utility of output, δ. As in the case of complete information, this effect is smaller if $k = 1$, and disappears when $k = 1$ and $\lambda \to 0$.

Lemma 4.1.

The output and the effort, $q^*(\hat{\beta})$ and $e^*(\hat{\beta})$, increase with the bureaucratic bias, δ.

Proof. See Appendix B.1. □

Again, the bureaucratic bias increases social welfare whenever the expected output level is larger than the reference output.

Lemma 4.2.

The expected social welfare, W^*, increases (decreases) with the bureaucratic bias, δ, if and only if the expected output, $\int_{\underline{\beta}}^{\overline{\beta}} q^*(\hat{\beta})\, d\hat{\beta}$, is larger (lower) than the reference output, q_{ref}.

Proof. See Appendix B.1. □

We conclude that, under adverse selection and moral hazard, the effects of a bureaucratic bias towards higher output are qualitatively the same as in the case of complete information.

Regarding the optimal contract, we find that the power of the optimal incentive scheme, that is, the fraction of the cost that is supported by the firm, K^*, is increasing with the bureaucratic bias, δ, for any value of the intrinsic marginal cost, $\hat{\beta}$ (if ψ''' is small enough).

The bureaucratic bias is shown to countervail both the adverse selection and the moral hazard problem. Firms tend to announce a high intrinsic cost (adverse selection) in order to achieve the required efficiency with a low effort. Since the government assigns a lower output to firms which announce a high intrinsic cost, this tendency is weaker in bureaucratic firms. Bureaucratic behavior leads to a more powered incentive scheme because the output is higher (and, therefore, the benefits associated with effort are greater) and also because of this reduction in the problem of adverse selection. To understand this second effect, notice that as the power of the incentive scheme increases, the gain associated with an observed cost that is lower than expected increases. Therefore, the firm has an increased tendency to announce a high intrinsic cost (in order to lower the government's expectations). Adverse selection limits, in this sense, the power of the incentive scheme. The problem of moral hazard is, therefore, also mitigated by bureaucratic behavior.

Lemma 4.3.

For small enough ψ''', the fraction of cost that is supported by the firm, $K^*(\hat{\beta})$, is increasing with δ. That is: $\dfrac{dK^*(\hat{\beta})}{d\delta} > 0$.

Proof. See Appendix B.1. □

The role of ψ''' is more subtle. From the incentive compatibility condition (6), a higher ψ'' implies that the information rent varies more with $\hat{\beta}$. The problem of adverse selection is, therefore, aggravated. This is why the power of the incentive scheme, K^*, is decreasing with ψ''. A stronger bureaucratic bias induces a higher effort, and, if ψ''' is large, an increase in ψ'' that dominates the previously mentioned effects, leading to a less powered incentive scheme.

5 The cost reimbursement payment system

In the case in which there is no cost-reducing effort ($C = \hat{\beta}q + \epsilon$), the government may propose the financing system known as cost-reimbursement, which consists in: (i) compensating the firm for all the costs which it incurs; plus (ii) a net payment in advance, $t(\beta)$, which does not depend on observed cost, and can even be negative because a bureaucratic firm enjoys producing a high output.

The utility of the firm ($U = t(\beta) + \delta\left[q(\beta) - q_{ref}\right]$) is independent of $\hat{\beta}$. Therefore, it must be constant across β for the firm to tell the truth (as lying does not yield a strictly positive gain). Hence, the government will choose $t(\beta)$ and $q(\beta)$ such that $U(\beta) = 0$, for any announcement β (the participation constraint is binding).

To produce a level of output that is lower than q_{ref}, the firm requires a positive net payment to participate, $t(\hat{\beta}) > 0$. If the output is higher than q_{ref}, the firm accepts a negative transfer, $t(\hat{\beta}) < 0$. Then, $\forall \hat{\beta} \in \left[\underline{\beta}, \overline{\beta}\right]$:

$$U(\hat{\beta}) = 0 \;\Leftrightarrow\; t(\hat{\beta}) = -\delta\left[q(\hat{\beta}) - q_{ref}\right]. \tag{9}$$

Observe that the bureaucratic bias, δ, leads to a significant difference with respect to the usual reimbursement payment (Laffont and Tirole, 1993), which is characterized by a null net transfer. Here, the net transfer (which may be interpreted as an advance payment, independent of the actual cost) may be positive or negative.

The government's problem is:

$$\max_{q(\hat{\beta}),t(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - (1+\lambda)\left[C(\hat{\beta}) + t(\hat{\beta})\right] + t(\hat{\beta}) + \delta\left[q(\hat{\beta}) - q_{ref}\right] - \\ -k\delta\left[q(\hat{\beta}) - q_{ref}\right] \, d\hat{\beta} \tag{10}$$

subject to

$$t(\hat{\beta}) = -\delta \left[q(\hat{\beta}) - q_{ref} \right]. \tag{9}$$

Equivalently:

$$\max_{q(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - (1+\lambda)\left\{\hat{\beta}q(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right]\right\} - k\delta\left[q(\hat{\beta}) - q_{ref}\right] \, d\hat{\beta}. \tag{11}$$

The first order condition of problem (11) is:

$$S'\left[q(\hat{\beta})\right] = (1+\lambda)\hat{\beta} - (1+\lambda-k)\delta. \tag{12}$$

It implies that output is increasing with the bureaucratic bias.

The second order condition is verified because $S''(q) < 0$.

As in the case of the optimal incentive scheme, expected social welfare increases (decreases) with the bureaucratic bias whenever the expected output level is higher (lower) than the reference output.[8]

The effects of the bureaucratic bias are the same and occur for the same reasons as in the case of complete information and of the optimal incentive scheme. There is a direct effect which results from the increase in the marginal social benefit of output associated with δ, and there is an indirect effect related to the decrease of the tax distortion which results from compensating the firm with an increase in output instead of an increase in the money transfer.

6 The prospective payment system

The prospective payment system consists of a fixed payment, $g(\beta)$, independent of the observed cost.[9] The net monetary transfer is: $g(\beta) - C(\hat{\beta}, e, q, \epsilon) = g(\beta) - (\hat{\beta} - e)q(\beta) - \epsilon$.

[8]Applying the Envelope Theorem, we obtain $\frac{dW^*}{d\delta} = (1+\lambda-k)\int_{\underline{\beta}}^{\overline{\beta}} (q^* - q_{ref}) d\hat{\beta}$.

[9]The prospective payment system is used, in some countries, in contracts between governments and hospitals for the provision of health care services. A fixed payment is attributed, based on the Diagnosis-Related Group (DRG) of an hospital's admission record, independently of the costs that the hospital comes to incur in.

Given the true value of the intrinsic marginal cost, $\hat{\beta}$, and the announced value, β, the firm chooses the effort level, e, that maximizes expected utility:

$$U(\beta,\hat{\beta}) = \max_{e}\left\{g(\beta) - (\hat{\beta} - e)q(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi(e)\right\}.$$

The first order condition with respect to e is:

$$\psi'\left[e(\beta)\right] = q(\beta). \tag{13}$$

Since the government's transfer does not depend on the observed cost, the relationship between effort and output level is the same as in the case of complete information.

The firm truthfully announces its intrinsic marginal cost if and only if:

$$\hat{\beta} \in arg \max_{\beta\in\left[\underline{\beta},\overline{\beta}\right]}\left\{g(\beta) - \left[\hat{\beta} - e(\beta)\right]q(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta)\right]\right\}.$$

The first order incentive compatibility constraint is $V'(\hat{\beta}) = -q(\hat{\beta})$, and the second order constraint is $q'(\hat{\beta}) < 0$. Therefore, for the firm to participate, it is sufficient that $V(\overline{\beta}) = 0$.

The objective of the government is to maximize expected social welfare:

$$\begin{aligned}\max_{q(\hat{\beta}),e(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} & S\left[q(\hat{\beta})\right] - (1+\lambda)g(\hat{\beta}) + g(\hat{\beta}) - \left[\hat{\beta} - e(\beta)\right]q(\beta) + \\ & +\delta\left[q(\hat{\beta}) - q_{ref}\right] - \psi\left[e(\hat{\beta})\right] - k\delta\left[q(\hat{\beta}) - q_{ref}\right]d\hat{\beta}\end{aligned} \tag{14}$$

subject to

$$\begin{aligned} V(\overline{\beta}) &= 0, \\ V'(\hat{\beta}) &= -q(\hat{\beta}), \\ q'(\hat{\beta}) &< 0. \end{aligned} \tag{15}$$

This problem is solved in Appendix A.3.

The conclusions obtained in the previous scenarios are maintained. The more efficient and more bureaucratic is the firm, the higher are the output and the effort (see Appendices A.3 and B.2).

The firm tends to announce a low efficiency for the government to transfer a high prospective payment. Since the government also selects a lower output, bureaucratic behavior counterbalances this tendency, alleviating the problem of adverse selection.

We also find that the expected social welfare increases (decreases) with the bureaucratic bias whenever the expected output level is larger (lower) than the reference output, as with the previous financing systems (see Appendix B.2).

7 Concluding remarks

We have studied procurement contracts between the government (principal) and a bureaucratic firm (agent), in the presence of moral hazard and adverse selection. Three different payment systems were considered: the optimal incentive scheme, cost reimbursement and prospective payment. In any case, with a bureaucratic provider, we observe a higher level of public good provision. Under the prospective payment and the optimal incentive schemes, the effort level is also higher. The optimal incentive scheme is shown to remain linear in observed cost but becomes more powered (the firm supports a higher fraction of the costs) when the manager is more bureaucratic. Welfare improves if and only if the expected output is higher than the output reference level of the managers.

The bureaucratic bias is shown to attenuate the adverse selection and moral hazard problem. Firms' tendency to announce a high intrinsic cost diminishes in bureaucratic firms (because the government would select a lower output). We have explained why this allows a more powered incentive scheme (a lower fraction of the costs is reimbursed), which reduces the problem of moral hazard.

The value of the manager's marginal utility of output is, in the present model, known by the regulator. It would be interesting to account for the fact that it is more likely to be his/her private information and to analyze the resulting equilibrium in such a framework. This will be the subject of further research.

A Appendix: Solving the model

A.1 The case of complete information

Using (1), problem (2) can be written as:

$$\max_{q,e,U} \left\{ S(q) - (1+\lambda)\left[U - \delta(q - q_{ref}) + \psi(e) + (\hat{\beta} - e)q\right] + U - k\delta(q - q_{ref})\right\} \tag{16}$$

subject to

$$U \geq 0.$$

With $\lambda > 0$, the participation constraint is binding ($U = 0$).

The first order conditions of problem (16) are:

$$S'(q) = (1+\lambda)\left(\hat{\beta} - e - \delta\right) + k\delta, \tag{17}$$

$$\psi'(e) = q. \tag{18}$$

The second order conditions of problem (16) are:[10]

$$\begin{aligned}
&S''(q) < 0,\\
&\psi''(e) > 0,\\
&S''(q)\psi''(e) + (1+\lambda) < 0.
\end{aligned}$$

By inspection of the first order conditions (17) and (18), we find that output, q_c^*, and effort, e_c^*, are decreasing functions of $\hat{\beta}$ and increasing functions of δ.

To see how the social welfare, W_c^*, varies with the bureaucratic bias, δ, apply the Envelope Theorem:

$$\frac{dW_c^*}{d\delta} = \left.\frac{\partial W_c}{\partial \delta}\right|_{q=q_c^*} = (1+\lambda-k)(q_c^* - q_{ref}).$$

A.2 The optimal incentive scheme

A.2.1 The problem of the firm

We start by analyzing the case in which there is no cost disturbance ($\epsilon = 0$).

Truthful behavior implies a total cost given by $C(\beta) = [\beta - e(\beta)]q(\beta)$. If the observed cost, $C = [\hat{\beta} - e]q(\beta)$, is different, the government imposes an extreme penalty to the firm (knife-edge mechanism):

$$C \neq C(\beta) \;\Rightarrow\; t(\beta, C) = -\infty.$$

[10] Corresponding to $\frac{\partial^2 f}{\partial q^2} < 0$, $\frac{\partial^2 f}{\partial e^2} < 0$ and $\frac{\partial^2 f}{\partial q^2}\frac{\partial^2 f}{\partial e^2} < \left(\frac{\partial^2 f}{\partial q \partial e}\right)^2$.

Still, a firm with cost $\hat{\beta}$ can claim to have a higher cost, $\beta > \hat{\beta}$, and choose a lower level of effort, $e < e(\beta)$, such that $C = C(\beta)$. In this case, the firm's deviation is concealed. Such level of effort is $e(\beta, \hat{\beta}) = e(\beta) + \hat{\beta} - \beta$, and the resulting net transfer is $s(\beta) = t\left[\beta, C(\beta)\right]$.

For any $\hat{\beta} \in \left[\underline{\beta}, \overline{\beta}\right]$, truthful revelation must maximize the utility of the firm (3):

$$\hat{\beta} \in \underset{\beta \in \left[\underline{\beta}, \overline{\beta}\right]}{\operatorname{argmax}}\, U(\beta, \hat{\beta}) = \underset{\beta \in \left[\underline{\beta}, \overline{\beta}\right]}{\operatorname{argmax}} \left\{ s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta) + \hat{\beta} - \beta\right] \right\}. \tag{19}$$

Let $V(\hat{\beta})$ be the value function of the firm's maximization problem:

$$V(\hat{\beta}) = \max_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} U(\beta, \hat{\beta}) = \max_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} \left\{ s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta) + \hat{\beta} - \beta\right] \right\}.$$

From the Envelope Theorem, we obtain the first order incentive compatibility constraint:[11]

$$V'(\hat{\beta}) = -\psi'\left[e(\hat{\beta})\right]. \tag{6}$$

The local second order condition, $\left.\dfrac{\partial^2 U(\beta, \hat{\beta})}{\partial \beta^2}\right|_{\beta = \hat{\beta}} < 0$, holds if (see Supp. Material C.1):

$$e'(\hat{\beta}) < 1. \tag{7}$$

The incentive compatibility condition (19) is equivalent to the first and second order conditions, (6) and (7).[12]

A.2.2 The problem of the government

We start by studying the following relaxed problem in which the second order incentive compatibility condition (7) is ignored (later, we shall check that this condition holds):

$$\max_{q(\hat{\beta}), e(\hat{\beta}), V(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - (1+\lambda)\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \psi\left[e(\hat{\beta})\right]\right\} -$$
$$-(1+\lambda)\left[\hat{\beta} - e(\hat{\beta})\right] q(\hat{\beta}) + V(\hat{\beta}) - k\delta\left[q(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta} \tag{20}$$

[11] The incentive compatibility condition (19) implies that the effort function, $e(\beta)$, and the utility function, $V(\hat{\beta})$, are differentiable almost everywhere (see Supp. Material C.2).

[12] See Laffont and Tirole (1986) or Supp. Material C.3.

subject to

$$V(\overline{\beta}) = 0, \tag{21}$$

$$V'(\hat{\beta}) = -\psi' \left[e(\hat{\beta}) \right]. \tag{6}$$

This is an optimal control problem with state variable $V(\hat{\beta})$ and control variables $e(\hat{\beta})$ and $q(\hat{\beta})$. The first order conditions are:

$$V(\overline{\beta}) = 0, \tag{21}$$

$$V'(\hat{\beta}) = -\psi' \left[e(\hat{\beta}) \right], \tag{6}$$

$$S' \left[q(\hat{\beta}) \right] = (1+\lambda) \left[\hat{\beta} - e(\hat{\beta}) - \delta \right] + k\delta, \tag{22}$$

$$\psi' \left[e(\hat{\beta}) \right] = q(\hat{\beta}) - \frac{\lambda}{1+\lambda} (\hat{\beta} - \underline{\beta}) \psi'' \left[e(\hat{\beta}) \right]. \tag{23}$$

This relaxed problem (20) has a unique interior optimum (see Supp. Material C.4).

To check that the omitted condition (7) is satisfied, differentiate equations (22) and (23) with respect to $\hat{\beta}$ (we use simplified notation, below, for simplicity of exposition):

$$\begin{cases} S'' q^{*'} = (1+\lambda)(1 - e^{*'}) \\ \psi'' e^{*'} = q^{*'} - \frac{\lambda}{1+\lambda} \left[\psi'' + (\hat{\beta} - \underline{\beta}) \psi''' e^{*'} \right] \end{cases} \Leftrightarrow \begin{cases} q^{*'} = \frac{1+\lambda}{S''} (1 - e^{*'}) \\ e^{*'} = \frac{1+\lambda - \left(\frac{\lambda}{1+\lambda} \right) S'' \psi''}{S'' \psi'' + 1 + \lambda + \frac{\lambda}{1+\lambda} (\hat{\beta} - \underline{\beta}) S'' \psi'''}. \end{cases} \tag{24}$$

Using Assumption 1, we find that $e^{*'} < 0$ and $q^{*'} < 0$, implying that (7) is verified. The solution of the relaxed problem (20) also solves the problem of the government (4).

The equilibrium transfer is such that:

$$t^*(\hat{\beta}) = V(\hat{\beta}) - \delta \left[q^*(\hat{\beta}) - q_{ref} \right] + \psi \left[e^*(\hat{\beta}) \right].$$

Integrating (6), we obtain:

$$V(\hat{\beta}) = V(\overline{\beta}) + \int_{\hat{\beta}}^{\overline{\beta}} \psi' \left[e(\gamma) \right] d\gamma.$$

Then:

$$t^*(\hat{\beta}) = \int_{\hat{\beta}}^{\overline{\beta}} \psi' \left[e^*(\gamma) \right] d\gamma - \delta \left[q^*(\hat{\beta}) - q_{ref} \right] + \psi \left[e^*(\hat{\beta}) \right].$$

If k increases, output and effort decrease, and the utility of the firm also decreases. From (23), we find that if ψ''' is sufficiently small, $q^*(\hat{\beta})$ and $e^*(\hat{\beta})$ move in the same direction. From (22), $q^*(\hat{\beta})$ and $e^*(\hat{\beta})$ must be decreasing when k increases. From (21) and (6) together with convexity of ψ, the decrease in $e^*(\hat{\beta})$ implies a decrease in $V^*(\hat{\beta})$.

A.3 The prospective payment scheme

We solve the following relaxed problem obtained by dropping (6) and then checking that the solution satisfies this constraint.

$$\max_{q(\hat{\beta}),e(\hat{\beta}),V(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - (1+\lambda)\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \psi\left[e(\hat{\beta})\right]\right\} -$$
$$-(1+\lambda)\left[\hat{\beta} - e(\hat{\beta})\right] q(\hat{\beta}) + V(\hat{\beta}) - k\delta\left[q(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta} \tag{25}$$

subject to

$$V(\overline{\beta}) = 0, \tag{15}$$
$$V'(\hat{\beta}) = -q(\hat{\beta}), \tag{6}$$

This is an optimal control problem with state variable $V(\hat{\beta})$ and control variables $e(\hat{\beta})$ and $q(\hat{\beta})$. The first order conditions are (see Supp. Material C.7):

$$V(\overline{\beta}) = 0, \tag{15}$$
$$V'(\hat{\beta}) = -q(\hat{\beta}), \tag{6}$$
$$\psi'\left[e(\hat{\beta})\right] = q(\hat{\beta}), \tag{13}$$
$$S'\left[q(\hat{\beta})\right] = (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \delta\right] + k\delta + \lambda(\hat{\beta} - \underline{\beta}). \tag{26}$$

The first order conditions are sufficient, under Assumption 1 (ii) (see Supp. Material C.7).

We can verify that the more efficient is the firm, the higher are the output and the effort, by differentiating equations (26) and (13):

$$\begin{cases} S'' q_p^{*'} = (1+\lambda)(1 - e_p^{*'}) + \lambda \\ \psi'' e_p^{*'} = q_p^{*'} \end{cases} \Leftrightarrow \begin{cases} q_p^{*'} = \psi'' \frac{1+2\lambda}{\psi'' S'' + 1 + \lambda} \\ e_p^{*'} = \frac{1+2\lambda}{\psi'' S'' + 1 + \lambda}. \end{cases}$$

We find that $e^{*'} < 0$ and $q^{*'} < 0$ which implies that (6) is verified. The solution of the relaxed problem (25) is the solution of the fully constrained problem (14).

The equilibrium transfer is such that $t_p^*(\hat{\beta}) = V_p^*(\hat{\beta}) - \delta\left[q_p^*(\beta) - q_{ref}\right] + \psi\left[e_p^*(\beta)\right]$, therefore:

$$t_p^*(\hat{\beta}) = \int_{\hat{\beta}}^{\overline{\beta}} q_p^*(\xi)\, d\xi - \delta\left[q_p^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e_p^*(\hat{\beta})\right].$$

B Appendix: Effect of the bureaucratic bias

B.1 The optimal incentive scheme

Proof of Lemma 4.1.

Differentiating equations (22) and (23), in order to δ we obtain:

$$\begin{cases} S'' \frac{dq^*(\hat{\beta})}{d\delta} = -(1+\lambda)\left(\frac{de^*(\hat{\beta})}{d\delta} + 1\right) + k \\ \psi'' \frac{de^*(\hat{\beta})}{d\delta} = \frac{dq^*(\hat{\beta})}{d\delta} - \left(\frac{\lambda}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi''' \frac{de^*(\hat{\beta})}{d\delta} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{dq^*(\hat{\beta})}{d\delta} = -\frac{1+\lambda}{S''}\left(\frac{de^*(\hat{\beta})}{d\delta} + 1\right) + \frac{k}{S''} \\ \frac{de^*(\hat{\beta})}{d\delta} = \frac{-\frac{1+\lambda}{S''} + \frac{k}{S''}}{\psi'' + \frac{1+\lambda}{S''} + \left(\frac{\lambda}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi'''} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{dq^*(\hat{\beta})}{d\delta} = \frac{1+\lambda}{S''}\left[\frac{1+\lambda-k}{\psi'' S'' + 1 + \lambda + S''\left(\frac{\lambda}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi'''} - 1\right] \\ \frac{de^*(\hat{\beta})}{d\delta} = -\frac{1+\lambda-k}{\psi'' S'' + 1 + \lambda + S''\left(\frac{\lambda}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi'''} \end{cases}.$$

Using Assumption 1 (i) and (ii), we find that $\frac{de^*(\hat{\beta})}{d\delta} > 0$ and $\frac{dq^*(\hat{\beta})}{d\delta} > 0$ (with $0 \leq k \leq 1$).

□

Proof of Lemma 4.2.

The expected social welfare function is given by:

$$W^* = \int_{\underline{\beta}}^{\overline{\beta}} S\left[q^*(\hat{\beta})\right] - (1+\lambda)\left\{-\delta\left[q^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e^*(\hat{\beta})\right] + \left[\hat{\beta} - e^*(\hat{\beta})\right] q^*(\hat{\beta})\right\} + \\ -\lambda V^*(\hat{\beta}) - k\delta\left[q^*(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta}. \quad (27)$$

The problem is subject to $V'^{*}(\hat{\beta}) = -\psi'\left[e^{*}(\hat{\beta})\right]$. Integrating by parts:

$$\int_{\underline{\beta}}^{\overline{\beta}} V^{*}(\hat{\beta})d\hat{\beta} = \left[V^{*}(\hat{\beta})(\hat{\beta}-\underline{\beta})\right]_{\underline{\beta}}^{\overline{\beta}} + \int_{\underline{\beta}}^{\overline{\beta}} \psi'\left[e^{*}(\hat{\beta})\right](\hat{\beta}-\underline{\beta})\, d\hat{\beta}.$$

Since $V(\overline{\beta}) = 0$, we have $\left[V^{*}(\hat{\beta})(\hat{\beta}-\underline{\beta})\right]_{\underline{\beta}}^{\overline{\beta}} = 0$.

Rewriting the problem (27):

$$W^{*} = \int_{\underline{\beta}}^{\overline{\beta}} S\left[q^{*}(\hat{\beta})\right] - (1+\lambda)\left\{-\delta\left[q^{*}(\hat{\beta}) - q_{ref}\right] + \psi\left[e^{*}(\hat{\beta})\right] + \left[\hat{\beta} - e^{*}(\hat{\beta})\right]q^{*}(\hat{\beta})\right\} + \\ -\lambda\psi'\left[e^{*}(\hat{\beta})\right](\hat{\beta}-\underline{\beta}) - k\delta\left[q^{*}(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta}.$$

Using the Envelope Theorem we find:

$$\frac{dW^{*}}{d\delta} = (1+\lambda-k)\int_{\underline{\beta}}^{\overline{\beta}}\left[q^{*}(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta}.$$

□

Proof of Lemma 4.3.

We know that both $e^{*}(\hat{\beta})$ and $q^{*}(\hat{\beta})$ are increasing in δ. Using equation (23) we can rewrite (8) as:

$$K^{*}(\hat{\beta}) = 1 - \frac{\left(\frac{\lambda}{1+\lambda}\right)(\hat{\beta}-\underline{\beta})\psi''\left[e^{*}(\hat{\beta})\right]}{q^{*}(\hat{\beta})}.$$

Therefore:

$$\frac{dK^{*}(\hat{\beta})}{d\delta} = -\left(\frac{\lambda}{1+\lambda}\right)(\hat{\beta}-\underline{\beta})\frac{\psi'''\left[e^{*}(\hat{\beta})\right]\frac{de^{*}(\hat{\beta})}{d\delta}q^{*}(\hat{\beta}) - \frac{dq^{*}(\hat{\beta})}{d\delta}\psi''\left[e^{*}(\hat{\beta})\right]}{q^{*}(\hat{\beta})^{2}}.$$

Thus, $\frac{dK^{*}(\hat{\beta})}{d\delta} > 0$ if and only if:

$$\psi'''\left[e^{*}(\hat{\beta})\right]\frac{de^{*}(\hat{\beta})}{d\delta}q^{*}(\hat{\beta}) - \frac{dq^{*}(\hat{\beta})}{d\delta}\psi''\left[e^{*}(\hat{\beta})\right] < 0 \Leftrightarrow \frac{\frac{dq^{*}(\hat{\beta})}{d\delta}}{\frac{de^{*}(\hat{\beta})}{d\delta}} > \frac{\psi'''\left[e^{*}(\hat{\beta})\right]q^{*}(\hat{\beta})}{\psi''\left[e^{*}(\hat{\beta})\right]}.$$

From the expressions for $\frac{dq^*(\hat{\beta})}{d\delta}$ and $\frac{de^*(\hat{\beta})}{d\delta}$ in Lemma 7, the condition above is always true when ψ''' is null. Therefore, we may conclude that, in this case, $K^*(\hat{\beta})$ is increasing in δ for any value of $\hat{\beta}$.

□

B.2 The prospective payment system

Lemma B.1. *The output, $q_p^*(\hat{\beta})$, and the effort, $e_p^*(\hat{\beta})$, are increasing in δ.*

Proof.

Differentiating equations (26) and (13), in order to δ we obtain:

$$\begin{cases} S''\frac{dq_p^*(\hat{\beta})}{d\delta} = -(1+\lambda)\left(\frac{de_p^*(\hat{\beta})}{d\delta}+1\right)+k \\ \psi''\frac{de_p^*(\hat{\beta})}{d\delta} = \frac{dq_p^*(\hat{\beta})}{d\delta} \end{cases} \Leftrightarrow \begin{cases} \frac{dq_p^*(\hat{\beta})}{d\delta} = \frac{\psi''(k-1-\lambda)}{S''\psi''+1+\lambda}, \\ \frac{de_p^*(\hat{\beta})}{d\delta} = \frac{k-1-\lambda}{S''\psi''+1+\lambda}. \end{cases}$$

Using Assumption 1 (i) and (ii), we find that $\frac{de^*(\hat{\beta})}{d\delta} > 0$ and $\frac{dq^*(\hat{\beta})}{d\delta} > 0$ with $0 \leq k \leq 1$.

□

Lemma B.2. *The expected social welfare, W_p^*, increases (decreases) with the bureaucratic bias, δ, whenever the expected output level, $\int_{\underline{\beta}}^{\overline{\beta}} q^*(\hat{\beta})d\hat{\beta}$, is larger (lower) than the reference output, q_{ref}.*

Proof.

The expected social welfare function can be written as:

$$\begin{aligned} W_p^* = \int_{\underline{\beta}}^{\overline{\beta}} & S\left[q_p^*(\hat{\beta})\right] - (1+\lambda)\left\{\psi\left[e_p^*(\hat{\beta})\right] - \delta\left[q_p^*(\hat{\beta}) - q_{ref}\right]\right\} - \\ & -(1+\lambda)\left[\hat{\beta} - e_p^*(\hat{\beta})\right] q_p^*(\hat{\beta}) - \lambda V_p^*(\hat{\beta}) - k\delta\left[q_p^*(\hat{\beta}) - q_{ref}\right] d\hat{\beta}. \end{aligned} \tag{28}$$

The problem is subject to $V_p'^*(\hat{\beta}) = -q_p^*(\hat{\beta})$. Integrating by parts:

$$\int_{\underline{\beta}}^{\overline{\beta}} V_p^*(\hat{\beta}) = \left[V_p^*(\hat{\beta})(\hat{\beta} - \underline{\beta})\right]_{\underline{\beta}}^{\overline{\beta}} + \int_{\underline{\beta}}^{\overline{\beta}} q_p^*(\hat{\beta})(\hat{\beta} - \underline{\beta})\, d\hat{\beta}.$$

Since $V_p^*(\overline{\beta}) = 0$, we have $\left[V^*(\hat{\beta})(\hat{\beta} - \underline{\beta})\right]_{\underline{\beta}}^{\overline{\beta}} = 0$.

Rewriting the problem (28):

$$W_p^* = \int_{\underline{\beta}}^{\overline{\beta}} S\left[q_p^*(\hat{\beta})\right] - (1+\lambda)\left\{-\delta\left[q_p^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e_p^*(\hat{\beta})\right] + \left[\hat{\beta} - e_p^*(\hat{\beta})\right] q_p^*(\hat{\beta})\right\} + \\ -\lambda q_p^*(\hat{\beta})(\hat{\beta} - \underline{\beta}) - k\delta\left[q_p^*(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta}.$$

Using the Envelope Theorem we find:

$$\frac{dW_p^*}{d\delta} = (1 + \lambda - k)\int_{\underline{\beta}}^{\overline{\beta}} \left[q_p^*(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta}.$$

□

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C Supplementary Material

C.1 Local second order condition

The local second-order condition of the maximization program, $\left.\frac{\partial^2 U(\beta,\hat{\beta})}{\partial\beta^2}\right|_{\beta=\hat{\beta}} < 0$, is equivalent to:

$$s''(\hat{\beta}) + \delta q''(\hat{\beta}) - \psi''[e(\hat{\beta})][e'(\hat{\beta}) - 1]^2 - \psi'[e(\hat{\beta})]e''(\hat{\beta}) < 0. \tag{29}$$

We want to show that it is equivalent to $e'(\hat{\beta}) < 1$.

Start by noticing that, with the incentive compatibility condition (19) being satisfied, the value function is:

$$V(\hat{\beta}) = s(\hat{\beta}) + \delta\left[q(\hat{\beta}) - q_{ref}\right] - \psi\left[e(\hat{\beta})\right]. \tag{30}$$

Evaluating the derivative of (30) and equating to (6), we obtain:

$$s'(\hat{\beta}) + \delta q'(\hat{\beta}) - \psi'\left[e(\hat{\beta})\right][e'(\hat{\beta}) - 1] = 0. \tag{31}$$

The derivative of (31) is:

$$s''(\hat{\beta}) + \delta q''(\hat{\beta}) - \psi''\left[e(\hat{\beta})\right]e'(\hat{\beta})\left[e'(\hat{\beta}) - 1\right] - \psi'\left[e(\hat{\beta})\right]e''(\hat{\beta}) = 0. \tag{32}$$

Subtracting (32) from (29), the local second order condition becomes:

$$-\psi''\left[e(\hat{\beta})\right]\left[-e'(\hat{\beta}) + 1\right] < 0.$$

Since $\psi'' > 0$, the local second order is verified if and only if:

$$e'(\hat{\beta}) < 1.$$

C.2 Differentiability of effort, transfer and utility

Proposition 2.

If deviations in the firm's concealment set are not profitable (19), then the effort function, $e(\beta)$, and the utility function, $V(\hat{\beta})$, are differentiable almost everywhere.

The proof of Proposition 2 is divided in the four claims, below.

Claim C.1.

$\beta < \hat{\beta} \Rightarrow e(\beta, \hat{\beta}) \geq e(\hat{\beta}, \hat{\beta})$.

Proof.

From the incentive compatibility constraints, we know that:

$$s(\hat{\beta}) + \delta \left[q(\hat{\beta}) - q_{ref}\right] - \psi \left[e(\hat{\beta}, \hat{\beta})\right] \geq s(\beta) + \delta \left[q(\beta) - q_{ref}\right] - \psi \left[e(\beta, \hat{\beta})\right]$$

and

$$s(\beta) + \delta \left[q(\beta) - q_{ref}\right] - \psi \left[e(\beta, \beta)\right] \geq s(\hat{\beta}) + \delta \left[q(\hat{\beta}) - q_{ref}\right] - \psi \left[e(\hat{\beta}, \beta)\right].$$

Adding the two inequalities, we obtain:

$$\psi \left[e(\beta, \hat{\beta})\right] - \psi \left[e(\beta, \beta)\right] \geq \psi \left[e(\hat{\beta}, \hat{\beta})\right] - \psi \left[e(\hat{\beta}, \beta)\right]. \tag{33}$$

Notice that, by definition:

$$e(\beta, \hat{\beta}) - e(\beta, \beta) = \hat{\beta} - \beta > 0$$

and

$$e(\hat{\beta}, \hat{\beta}) - e(\hat{\beta}, \beta) = \hat{\beta} - \beta > 0.$$

Since these differences coincide, strict convexity of ψ together with (33) implies that:

$$e(\beta, \hat{\beta}) \geq e(\hat{\beta}, \hat{\beta}).$$

□

Claim C.2.

$e(\beta, \hat{\beta})$ is nonincreasing in β.

Proof.

Let $\beta > \beta'$ and define $\Delta(\hat{\beta}) \equiv e(\beta', \hat{\beta}) - e(\beta, \hat{\beta})$. We want to prove that $\Delta(\hat{\beta}) \geq 0$.

Notice that: $\Delta(\hat{\beta}) = e(\beta') - \beta' - e(\beta) + \beta$. Thus $\Delta(\hat{\beta})$ does not depend on $\hat{\beta}$.

Then, $\Delta(\hat{\beta}) = \Delta(\beta) = e(\beta', \beta) - e(\beta, \beta)$.

By Lemma 1, $\Delta(\beta) \geq 0$.

$\square$

Since the effort level is not greater than $\overline{\beta}$, Claim C.2 implies that $e(\beta, \hat{\beta})$ is a.e. differentiable in β. Therefore, $e(\beta) = e(\beta, \hat{\beta}) + \beta - \hat{\beta}$ also is.

Claim C.3.

$U(\beta, \hat{\beta})$, as a function of β, is nondecreasing on $\left[\underline{\beta}, \hat{\beta}\right]$ and nonincreasing on $\left[\hat{\beta}, \overline{\beta}\right]$.

Proof.

Let us first show monotonicity on $\left[\underline{\beta}, \hat{\beta}\right]$. Assume that $\beta < \beta' < \hat{\beta}$ and, by way of contradiction, $U(\beta, \hat{\beta}) > U(\beta', \hat{\beta})$. Thus:

$$s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta, \hat{\beta})\right] > s(\beta') + \delta\left[q(\beta') - q_{ref}\right] - \psi\left[e(\beta', \hat{\beta})\right].$$

On the other hand, a firm with cost β' prefers to announce β' rather than announce β:

$$s(\beta') + \delta\left[q(\beta') - q_{ref}\right] - \psi\left[e(\beta', \beta')\right] \geq s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta, \beta')\right].$$

Adding the two equations, we obtain:

$$\psi\left[e(\beta, \beta')\right] - \psi\left[e(\beta', \beta')\right] > \psi\left[e(\beta, \hat{\beta})\right] - \psi\left[e(\beta', \hat{\beta})\right].$$

By definition:

$$e(\beta, \beta') - e(\beta', \beta') = e(\beta, \hat{\beta}) - e(\beta', \hat{\beta}).$$

From Lemma 1, $e(\beta, \beta') \geq e(\beta', \beta')$. Thus:

$$e(\beta, \beta') - e(\beta', \beta') = e(\beta, \hat{\beta}) - e(\beta', \hat{\beta}) \geq 0.$$

Since $e(\beta, \beta') < e(\beta, \hat{\beta})$ and ψ is convex, the last equation implies:

$$\psi\left[e(\beta, \beta')\right] - \psi\left[e(\beta', \beta')\right] > \psi\left[e(\beta, \hat{\beta})\right] - \psi\left[e(\beta', \hat{\beta})\right].$$

Which is a contradiction.

Monotonicity on $\left[\hat{\beta}, \bar{\beta}\right]$ can be proved in the same way.

$\square$

Claim C.4.

$s(\beta) + \delta q(\beta)$ is nonincreasing.

Proof.

By definition:

$$U(\beta, \underline{\beta}) = s(\beta) + \delta \left[q(\beta) - q_{ref}\right] - \psi \left[e(\beta, \underline{\beta})\right] \Leftrightarrow$$
$$\Leftrightarrow s(\beta) + \delta \left[q(\beta) - q_{ref}\right] = U(\beta, \underline{\beta}) + \psi \left[e(\beta, \underline{\beta})\right].$$

From Claim C.2: $\psi \left[e(\beta, \underline{\beta})\right]$ is nonincreasing with β.

From Claim C.3: $U(\beta, \underline{\beta})$ is nonincreasing with β.

Therefore, $s(\beta) + \delta q(\beta)$ must also be nonincreasing with β.

$\square$

Claims C.2 and C.4 imply that $e(\beta, \hat{\beta})$, $e(\beta)$ and $s(\beta) + \delta q(\beta)$ are a.e. differentiable.

Hence, $U(\beta, \hat{\beta}) = s(\beta) + \delta \left[q(\beta) - q_{ref}\right] - \psi \left[e(\beta, \hat{\beta})\right]$ and $V(\hat{\beta}) = s(\hat{\beta}) + \delta \left[q(\hat{\beta}) - q_{ref}\right] - \psi \left[e(\hat{\beta})\right]$ are also a.e. differentiable.

C.3 The local second order condition implies the global one

Proposition 3.

If the local second order condition, (7), holds, then the first order condition (6) is equivalent to the incentive compatibility condition (19).

Proof.

Recall that:

$$U(\beta, \hat{\beta}) = s(\beta) + \delta \left[q(\beta) - q_{ref}\right] - \psi \left[e(\beta, \hat{\beta})\right].$$

Then:

$$\frac{\partial^2 U}{\partial \beta \partial \hat{\beta}} = -\psi'' \left[e(\beta) + \hat{\beta} - \beta \right] \left[e'(\beta) - 1 \right].$$

Using strict convexity of ψ, we find that if the local second order condition is strictly satisfied, then $\frac{\partial^2 U}{\partial \beta \partial \hat{\beta}} > 0$.

Announcing the truth gives a local maximum for the firm of type $\hat{\beta}$. Suppose that there is another announcement, $\beta \neq \hat{\beta}$, that also satisfies the first order condition:

$$\frac{\partial U(\beta, \hat{\beta})}{\partial \beta} = \frac{\partial U(\hat{\beta}, \hat{\beta})}{\partial \beta} = 0.$$

This would imply that:

$$\frac{\partial U(\beta, \hat{\beta})}{\partial \beta} = \frac{\partial U(\beta, \beta)}{\partial \beta} = 0.$$

Which contradicts the fact that $\frac{\partial^2 U}{\partial \beta \partial \hat{\beta}} > 0$. The local maximum is a global maximum.

$\square$

C.4 Problem of the government

Necessary conditions

Proposition 4.

The following are necessary conditions for an interior optimum of problem (20):

$$V(\overline{\beta}) = 0, \tag{21}$$

$$V'(\hat{\beta}) = -\psi' \left[e(\hat{\beta}) \right], \tag{6}$$

$$S' \left[q(\hat{\beta}) \right] = (1 + \lambda) \left[\hat{\beta} - e(\hat{\beta}) - \delta \right] + k\delta, \tag{22}$$

$$\psi' \left[e(\hat{\beta}) \right] = q(\hat{\beta}) - \frac{\lambda}{1 + \lambda} (\hat{\beta} - \underline{\beta}) \psi'' \left[e(\hat{\beta}) \right]. \tag{23}$$

Proof.

Consider problem (20). The Hamiltonian is:

$$\begin{aligned} H = S\left[q(\hat{\beta})\right] - (1+\lambda)\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \psi\left[e(\beta)\right] + \left[\hat{\beta} - e(\hat{\beta})\right]q(\hat{\beta})\right\} \\ +V(\hat{\beta}) - k\delta\left[q(\hat{\beta}) - q_{ref}\right] + \mu\left\{-\psi'\left[e(\hat{\beta})\right]\right\}, \end{aligned} \tag{34}$$

where μ is the multiplier associated with (6). The Pontryagin principle[13] yields:

$$\frac{\partial H}{\partial q} = S'\left[q(\hat{\beta})\right] - (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \delta\right] - k\delta = 0,$$

$$\frac{\partial H}{\partial e} = -(1+\lambda)\left\{\psi'\left[e(\hat{\beta})\right] - q(\hat{\beta})\right\} - \mu\psi''\left[e(\hat{\beta})\right] = 0, \tag{35}$$

$$\mu'(\hat{\beta}) = -\frac{\partial H}{\partial V} = \lambda. \tag{36}$$

Furthermore, $\underline{\beta}$ is a free boundary so that:

$$\mu(\underline{\beta}) = 0. \tag{37}$$

Integrating (36) and using (37), we obtain:

$$\mu(\hat{\beta}) = \lambda(\hat{\beta} - \underline{\beta}).$$

Substituting in (35) above:

$$\psi'\left[e(\hat{\beta})\right] = q(\hat{\beta}) - \left(\frac{\lambda}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi''\left[e(\hat{\beta})\right].$$

□

Sufficient conditions

Proposition 5. *Problem (20) has a unique interior optimum.*

[13] See, for example, Chiang and Wainwright (2005).

Proof.

The second order derivatives of the Hamiltonian (34) are:

$$\begin{aligned}
\frac{\partial^2 H}{\partial q^2} &= S''\left[q(\hat{\beta})\right] < 0, \\
\frac{\partial^2 H}{\partial e^2} &= -(1+\lambda)\,\psi''\left[e(\hat{\beta})\right] + \left(\frac{\lambda}{1+\lambda}\right)\left(\hat{\beta} - \underline{\beta}\right)\psi'''\left[e(\hat{\beta})\right] < 0, \\
\frac{\partial^2 H}{\partial q \partial e} &= \frac{\partial^2 H}{\partial e \partial q} = 1+\lambda.
\end{aligned}$$

The determinant of the Hessian is:

$$\begin{aligned}
|H| = S''\left[q(\hat{\beta})\right]\left\{-(1+\lambda)\,\psi''\left[e(\hat{\beta})\right] + \left(\tfrac{\lambda}{1+\lambda}\right)\left(\hat{\beta} - \underline{\beta}\right)\psi'''\left[e(\hat{\beta})\right]\right\} - \\
-(1+\lambda)^2 \geq -S''\left[q(\hat{\beta})\right](1+\lambda)\,\psi''\left[e(\hat{\beta})\right] - (1+\lambda)^2.
\end{aligned}$$

Using Assumption 1 (ii), we find that $|H| > 0$. The first order conditions have a unique interior solution.

Finally, observe that the argument in Laffont and Tirole (1986, p.639) applies. Pontryagin's Principle requires V to be piecewise differentiable with a finite number of pieces, while we only know that V is a.e. differentiable and decreasing. The space of a.e. differentiable decreasing functions in $[\underline{\beta}, \overline{\beta}]$ is a closed and convex subset of the Banach space $L^\infty([\underline{\beta}, \overline{\beta}], \mathbb{R})$. Any decreasing function in $[\underline{\beta}, \overline{\beta}]$ that is a.e. differentiable can be approximated as closely as desired by a piecewise-continuous function. Therefore, the maximum in the subspace of piecewise-continuous functions (the solution that we found above) is the maximum in the general space of a.e. differentiable functions (the solution of the general problem).

□

C.5 Implementation

Proposition 1

Under Assumption 1, the optimal incentive compatible allocation, $[q^*(\hat{\beta}), e^*(\hat{\beta}), t^*(\hat{\beta})]$, can be implemented by a contract that is linear in observed cost:

$$t(\beta, C) = t^*(\beta) + K^*(\beta)\left[C^*(\beta) - C\right].$$

Proof.

A firm with intrinsic marginal cost $\hat{\beta}$ solves:

$$\max_{\beta,e} \; \{t^*(\beta) + E\{K^*(\beta)[C^*(\beta) - C]\} + \delta[q^*(\beta) - q_{ref}] - \psi(e)\}. \tag{38}$$

Notice that:

$$K^*(\beta)[C^*(\beta) - C] = \psi'[e^*(\beta)]\left[e - e^*(\beta) + \beta - \hat{\beta} - \frac{\epsilon}{q^*(\beta)}\right].$$

Substituting in (38), we obtain:

$$\max_{\beta,e} \; \left\{t^*(\beta) + \psi'[e^*(\beta)]\left[e - e^*(\beta) + \beta - \hat{\beta}\right] + \delta[q^*(\beta) - q_{ref}] - \psi(e)\right\}. \tag{39}$$

Optimization with respect to e yields:

$$\psi'[e^*(\beta)] = \psi'(e) \Leftrightarrow e = e^*(\beta).$$

Substituting again, problem (39) simplifies:

$$\max_{\beta} \; \left\{t^*(\beta) + \psi'[e^*(\beta)]\left(\beta - \hat{\beta}\right) + \delta[q^*(\beta) - q_{ref}] - \psi[e^*(\beta)]\right\}.$$

Optimizing with respect to β:

$$t^{*'}(\beta) + \psi''[e^*(\beta)]\,e^{*'}(\beta)(\beta - \hat{\beta}) + \psi'[e^*(\beta)] + \delta q^{*'}(\beta) - \psi'[e^*(\beta)]\,e^{*'}(\beta) = 0.$$

Substituting $\beta = \hat{\beta}$ we obtain:

$$t^{*'}(\hat{\beta}) + \psi'\left[e^*(\hat{\beta})\right]\left[1 - e^{*'}(\hat{\beta})\right] + \delta q^{*'}(\hat{\beta}) = 0.$$

Which is true as it coincides with the first order incentive compatibility condition (6).

Notice that the firm's second order condition for (39) is satisfied, as it boils down to:

$$e^{*'}(\beta) \leq 0. \tag{40}$$

The second order condition (40), which is stronger than (7), is necessary for this way of implementing the optimal solution, which requires the transfer to be linear in cost.

$\square$

C.6 Nonlinearity and cost disturbances

Let us show that a scheme that is not linear in cost cannot implement the optimal solution for all probability distributions of the cost disturbance.

We know that $t(\beta, C)$ must satisfy:

$$s^*(\beta) = Et\left\{\beta, [\beta - e^*(\beta)]q^*(\beta) + \epsilon\right\}.$$

If t is not linear in cost, there exist β, C_1, C_2 and C_3 such that

$$\frac{t(\beta, C_1) - t(\beta, C_2)}{C_1 - C_2} \neq \frac{t(\beta, C_1) - t(\beta, C_3)}{C_1 - C_3}.$$

Define $\epsilon_i \equiv C_i - [\beta - e^*(\beta)]q^*(\beta)$, and consider the family of discrete distributions with three atoms at e_1, e_2 and e_3 and no weight elsewhere (since these distributions can be approximated by continuous distributions, we could actually restrict ourselves to continuous distributions). It is clear that by varying the weights on the three disturbance levels and given the last equation, the first equation cannot always be satisfied.

C.7 The prospective payment scheme

Problem of the government - necessary conditions

Consider problem (25). The Hamiltonian is:

$$\begin{aligned} H = S\left[q(\hat{\beta})\right] - (1+\lambda)\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \psi\left[e(\beta)\right] + \left[\hat{\beta} - e(\hat{\beta})\right]q(\hat{\beta})\right\} + \\ + V(\hat{\beta}) - k\delta\left[q(\hat{\beta}) - q_{ref}\right] + \nu\left[-q(\hat{\beta})\right], \end{aligned} \tag{41}$$

where ν is the multiplier associated with (6). The Pontryagin principle yields:

$$\frac{\partial H}{\partial q} = S'\left[q(\hat{\beta})\right] - (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \delta\right] - k\delta - \nu = 0, \tag{42}$$

$$\frac{\partial H}{\partial e} = -(1+\lambda)\left\{\psi'\left[e(\hat{\beta})\right] - q(\hat{\beta})\right\} = 0,$$

$$\nu'(\hat{\beta}) = -\frac{\partial H}{\partial V} = \lambda. \tag{43}$$

Furthermore, $\underline{\beta}$ is a free boundary so that

$$\nu(\underline{\beta}) = 0. \tag{44}$$

Integrating (43) and using (44), we obtain

$$\nu(\hat{\beta}) = \lambda(\hat{\beta} - \underline{\beta}).$$

Substituting in (42) above:

$$S'\left[q(\hat{\beta})\right] = (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \delta\right] + k\delta + \lambda(\hat{\beta} - \underline{\beta}).$$

Sufficiency conditions

The second order derivatives of the Hamiltonian (41) are:

$$\begin{aligned}
\frac{\partial^2 H}{\partial q^2} &= S''\left[q(\hat{\beta})\right] < 0, \\
\frac{\partial^2 H}{\partial e^2} &= -(1+\lambda)\,\psi''\left[e(\hat{\beta})\right] < 0, \\
\frac{\partial^2 H}{\partial q \partial e} &= \frac{\partial^2 H}{\partial e \partial q} = 1+\lambda.
\end{aligned}$$

The determinant of the Hessian is:

$$\begin{aligned}
|H| &= -(1+\lambda)\,S''\left[q(\hat{\beta})\right]\psi''\left[e(\hat{\beta})\right] - (1+\lambda)^2. \\
|H| &> 0 \Leftrightarrow S''\left[q(\hat{\beta})\right]\psi''\left[e(\hat{\beta})\right] > 1+\lambda.
\end{aligned}$$

We find that $|H| > 0$, by Assumption 1 (ii).