Problem Set

1. Consider an economy with two financial assets: a bond, with a fixed interest rate, *r*; and a stock, which pays a constant flow of dividends, *d*. Assume that the interest and the dividends are paid in continuous time and that the agents have rational expectations regarding the evolution of the stock price.

a) Obtain the differential equation that describes the evolution of the stock price. Solve the differential equation and make a graphical representation of the solutions.

b) Between *t=0* and *t=4*, the flow of dividends and the interest rate were *d=0,3* and *r=0,05*, and the stock price remained constant. Consider the following unexpected announcements:

i) At *t=4,* it is announced that, starting from this moment, the flow of dividends will decrease to *d=0,2*.

ii) At *t=4*, it is announced that a tax rate of 20% on the flow of dividends will be introduced at *t=6*.

iii) At *t=4*, it is announced that the agents will receive a prize of *0,5 per* stock at *t=6*.

For each scenario, find the evolution of the stock price between *t=0* and *t=8*. Calculate, in particular, the stock price at *t=5.* Justify your answer and make a graphical representation of the results.

2. Consider an economy with two financial assets: a bond with a fixed interest rate, $r=0,05$; and a stock, which pays a dividend, d. Assume that the assets are traded in discrete time and that the interest and dividends are paid between the trading periods. Assume also that the agents have rational expectations regarding the evolution of the stock price.

a) Obtain the difference equation that describes the evolution of the stock price. Solve the difference equation and make a graphical representation of the solutions.

b) Suppose that the stock pays a dividend $d=1$ between $t=10^+$ and $t=19^+$ and no dividends in every other period. Find the evolution of the stock price.

c) Assume that traders are neutral with respect to risk and consider the following uncertainty about the evolution of dividends. There is a 50% chance that $d=1$ in every period and a 50% chance that $d=1$ until $t=9^+$ and $d=2$ from $t=10^+$ onwards. Find the evolution of the stock price.

i) Assuming that agents only discover the actual evolution of the dividends at $t=10^+$.

ii) Assuming that the actual evolution of the dividends is announced at $t=5^+$.

3. Consider the neoclassical model of economic growth. The production function of the economy is given by: $Y = K^{0,4} L^{0,6}$. The evolution of the labor supply is given by: $L(t) = e^{0,01t}$, while the capital accumulation follows a tradition according to which the output is allocated in equal shares for consumption and investment. It is known that, at $t_0=0$, the stock of capital was equal to *1* unit and that the depreciation rate is $\delta=0,09$.

a) Characterize the economy at $t_1 = ln(2)/0,06$.

b) At t_1, Ezequiel Valadas was democratically elected president, and has the possibility of choosing a new savings rate. His single objective is to choose the savings rate that maximizes the welfare of the population at the moment of the next elections (the memory of the electorate is very short). What should be his choice if the date of the elections is $t_2 = ln(2)/0,03$?

c) Which would be the choice of Ezequiel Valadas if $t_2 = ln(2)/0,01$? Explain (from an economic point of view) why the chosen savings rate depends on t_2. What happens when $t_2 \rightarrow +\infty$?

d) If Ezequiel Valadas could vary the savings rate at every moment (between t_1 e t_2), what would be his decision?

4. Consider an economy whose macroeconomic behavior is described by:

(IS) $Y'(t) = v_y\,(a_4 - a_1 r - Y)$

(LM) $r'(t) = v_r\,(a_5 + a_2 Y - a_3 r),$

where a_1, a_2, a_3 and a_4 are strictly positive constants and a_5 is another constant.

a) Find the macroeconomic equilibrium, and study whether it is stable or not.

b) Suppose that $a_1=a_2=a_3=a_4=1$ and $a_5=0$. Obtain the evolution of the economy, given that $Y(0)=1{,}5$ and $r(0)=3{,}5$:

i) with $v_y=1$ and $v_r=6$.

i**i)** with $v_y=1$ and $v_r=1$.

c) Supposing that the economy is initially in equilibrium, obtain the temporal evolution of Y and r that would result from an decrease in a_2 from 1 to 0.5, in each of the two previous scenarios. Represent graphically.

5. Consider the multiplier-accelerator model:

$C(t) = C_0 + c \cdot Y(t-1)$, with $0<c<1$;

$I(t) = I_0 + k \cdot [C(t) - C(t-1)]$, with $k>0$;

$Y(t) = C(t) + I(t)$.

a) Supposing that the economy is initially in equilibrium (stable), obtain the temporal evolution of output, consumption and investment that results from an increase in consumption (C_0 *becomes equal to* C_1). Represent graphically.