Welfare-Enhancing, Trade-Restricting Collusion in Geographically Separated Markets with Differentiated Goods*

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May 2009

Abstract

This paper examines a differentiated product duopoly operating in two geographically separate markets. Firms operate a single production facility each; facilities are located in different markets, but firms can supply both markets from their respective facility. Shipping across markets is costly; thus, every firm has a cost advantage in its home market. Consumers treat the two products as horizontally differentiated substitutes and their preferences are identical in both markets. In the non-cooperative equilibrium, each firm has a smaller market share and mark-up in the away market. The asymmetric mark-up leads to a market distortion, with more consumers than is socially optimal purchasing the imported good. Collusion between the two firms partially mitigates this distortion by reducing cross-hauling, resulting in higher aggregate welfare. In the absence of a cartel, governments can replicate the cartel's market allocation through a system of regulation, tariffs and subsidies, chosen in a way that raises both consumer welfare and firm profits.

JEL Codes: L13, L41, D43.

Keywords: Cross-hauling, Cartels, Coordinated Effects, Destructive Competition, Market Allocation Schemes, Home-market Principle, Strategic Trade, Two-way Trade.

*Preliminary and Incomplete. We would like to than Joe Farrell, Joe Harrington, Qihong Liu, Marco Ottaviani, Alexander Rasch, and seminar participants in the International Industrial Organization Conference for helpful comments. All errors are own own.

1 Introduction

In a typical collusive agreement, participants may coordinate on several dimensions besides setting a collusive price, such as deciding which geographic regions, customer types or product variants are to be supplied by the different constituent firms.[1] On the geographic dimension, Harrington (2006) points out that an element common to the market sharing scheme of a number of real world cartels is the adoption of a "home-market principle".[2] By this principle, each cartel member is given preference in supplying its home market—a region, say, where its production facilities are located—at the expense of supplying other regional markets.

An implication of such a market allocation scheme is that while "(i)n a competitive market, one would expect a rise in a firm's price, *ceteris paribus*, to result in more imports..., an allocation scheme based on the home-market principle would result in the combination of a higher price and *fewer* imports" (Harrington 2006, p.36, original emphasis). Examples of such cartels include choline chloride, lysine and methionine, where in the latter case "the home-market principle was, in fact, the instigating factor for cartel formation" (Harrington 2006, p.35-6). Along these lines, Röller and Steen (2006) examine an official Norwegian cement cartel, documenting the role of a common sales office whose "primary task was to organize sales in a better way, to prevent cross-transportation and unprofitable competition" (p.324)[3]. Strand (2002) looks at the European Union's sugar sector, in which national quotas under the Common Market Organization (CMO) appear to help firms allocate markets geographically; he cites a large sugar buyer saying that "(i)t is in every sugar supplier's best interest to stay out of each other's markets" (p.14). Similarly, Salvo (2007) studies the division of regional markets by firms in the Brazilian cement industry. One feature "spatial" industries such as cement and sugar share is the high cost of transport given their low value to weight ratio (Scherer et al 1975). Moreover, in a world of imperfect information, this type of geographic market sharing may facilitate the monitoring of each firm's supply as to ensure the implementation

[1] Such multi-dimensional cartel agreements were a feature of some of the earliest cartels (e.g., see Deltas, Serfes and Sicotte, 1999).

[2] See also Motta (2004), pages 187.

[3] Regarding the Norwegian cartel's relation with the broader European continent, Röller and Steen (2006) argue that "competition is a multimarket game where credible threats to enter each other's markets prevent firms from entering other countries" (p.324). Lommerud and Sørgard (2001) cite a scheme, uncovered by the European Commission in 1994, to limit intra-EU trade of cement (as well as the prosecution in the 1970s of Japanese and European synthetic fiber producers for agreeing to completely restrain exports to each other's markets). On how the ability to sell in multiple markets may facilitate collusion, see Bernheim and Whinston (1990).

of the collusive agreement.[4] [5]

When products are homogeneous, one might expect optimal cartel agreements to favor full geographic segmentation (as tends to be the case in the cement and sugar examples)—Pinto (1986), for example, models an international homogeneous-good cartel which does not trade in equilibrium. When products are differentiated, it may no longer be the case that the cartel would choose to divide markets geographically. Such a division entails a loss of total surplus, as the willingness of some consumers to pay for the product of one of the cartel members may be much lower than their willingness to pay for the products of other members. Therefore, such geographic restriction may reduce the surplus that the cartel can extract. Indeed, in such an environment, it is not clear whether and to what extent the cartel would restrict shipments across geographic markets. Despite the empirical relevance of geographic market division schemes, the literature has devoted limited attention to this topic. Hence, the answer to this and other related questions is not yet known.

In this paper we contribute towards closing this gap in the literature. We consider a horizontally differentiated perfect-monitoring duopoly model in a sufficiently simplified framework that permits the full characterization and comparison of the competitive (Nash) equilibrium, perfectly collusive, and socially optimal outcomes. There are two geographically separated markets, 1 and 2, each market being home to one firm: firm A is located in market 1 and firm B is located in market 2. To supply consumers in the other market, a firm incurs an additional linear trade cost per unit shipped. Within each market, consumers vary in their preferences over a differentiated good, each market being modeled as a uniform mass of consumers distributed over a unidimensional space of product characteristics, i.e. we embed Hotelling's unit interval in each market. Each of the two firms produces a single different variety: firm A's offering lies at the left endpoint of the unit interval and firm B's offering lies at the right endpoint. Completing the model, a consumer's disutility from consuming a variety other than her ideal variety is linear in the distance along this interval. To abstract away from aggregate demand effects and focus on strategic effects (see below), we parameterize the model such that each consumer chooses to purchase only one of the two goods.

[4] Motta (2004, p.141) highlights that market allocation schemes "have the advantage of allowing for prices to adjust to new demand and cost conditions without triggering price wars. A market allocation scheme avoids the possibility that, if a shock reduces production costs or market demand, a price reduction might trigger a price war. As long as each firm does not serve segments of demand (explicitly or tacitly) allocated to rivals, prices can change without the collusive outcome being disrupted. This probably explains why such collusive schemes are often used." The reduced occurrence of costly cartel-disciplining price wars à la Green and Porter (1984) which Motta alludes to, while relevant, lies outside the scope of this paper, and is left for future research.

[5] Casual (and unpublished) interviews by one of the authors with executives in a certain spatial industry suggest that the industry believes (or purports to believe) that a tacit understanding based on the home-market principle can enhance social welfare, in that resources are not wasted on cross-hauling—while admitting to the private benefit.

We derive equilibrium outcomes under two alternative market-based behavioral benchmarks: (i) a Nash equilibrium in prices—the "(imperfectly) competitive regime", and (ii) the joint-profit maximizing cartel—the "(fully) collusive regime"[6]. We first show that the competitive regime is characterized by more trade across geographic markets, as each competitive firm vies to sell to consumers whose tastes are closer to their product offering than that of the rival firm (and we have assumed no home bias so that, at equal prices, one half of consumers prefer the imported variety over the home variety). Relative to the competitive outcome, the (perfect) cartel cross-hauls less quantity, allocating a greater share of each market to the home firm's good and engaging in less price discrimination in favor of the imported good.

We next show that, at all positive trade cost levels, social welfare under collusion *exceeds* social welfare under competition. While the good each consumer chooses in the trade-prone competitive regime is at least as close to her ideal variety when compared to the collusive regime, this welfare effect in favor of the competitive regime is dominated by a lower cross-hauling cost in the collusive regime. This is an example of an industry where better meeting consumers' tastes for variety through trade competition may not be socially desirable. From the social viewpoint, competition leads to "excessive trade", with collusion serving as a mechanism to address this failure.[7] We then show that the correction mechanism is only partial: while lower than in the competitive outcome, there is *still too much trade in the collusive outcome* when compared to society's first-best outcome. Even more surprisingly, collusion can increase consumer surprlus relative to competition: some of the increase in welfare the collusion generates may be captured by the consumers.[8]

It is instructive to highlight the intuition for the welfare comparisons. Observe that from a welfare point of view, prices cancel out because they are a transfer. Only market allocations matter, and for these, what is important is relative prices (or relative cost-adjusted prices). Since markets and all equilibria are symmetric, we can consider welfare effects in a single market. In each market, the "home" firm has a cost advantage over the "away" firm because the home firm does not incur the shipping cost. Consider first the Nash equilibrium. In this equilibrium, the high-cost (away) firm

[6]We assume that this perfect cartel's agreement can be enforced by, say, grim trigger strategies à la Friedman (1971). This will in general be possible for firms that exhibit a suficiently high discount factor. See the appendix for a characterization.

[7]These results are reminiscent of, though conceptually distinct from, the suggestions that in some industries competitive markets can lead to "destructive competition" and that cartels can help stabilize them. For more discussion see Deltas, Sicotte and Tomczak (2008) and references therein.

[8]In that case, traditional damage from anti-competitive behavior calculations would show negative damages. When collusion leads to an increase in Consumer Surplus, it would increase the price of the imported good and decrease the price of the domestic good, but the quantity-weighted price could go down.

will choose a lower mark-up than the low-cost (home) firm, because the high-cost firm has a smaller market share and thus more aggressive pricing results in smaller revenue losses from the sales to inframarginal consumers. Therefore, the price difference between the two firms will be substantially lower than the cost difference (the trade cost incurred by the importing firm). As a result, the number of consumers who will be purchasing from the importing firm will be too high relative to the social optimum (a social planner would set the price difference between the two firms equal to the trade cost). Welfare would be higher if relative prices were to change so that some of the consumers purchasing from the high-cost supplier were to switch to the low-cost supplier. The cartel partially eliminates this distortion in an effort to increase total surplus so that it can appropriate some of this increase as higher profits.[9] The distortion is not fully eliminated because some price discrimination against those who purchase the home good—who in equilibrium constitute the majority of the market—increases the cartel's surplus. As a result, there still is welfare-reducing cross-hauling by the cartel. Relative to the social planner, the cartel takes more advantage of consumer heterogeneity in the preferences for the two goods. A social planner would raise the price of the imported good relative to the home good, no longer price discriminating against the home good, and thus will reduce cross-hauling even more than the cartel would. The possibility that consumers of a market exhibit higher average willingness to pay for the product produced in that market (i.e., the possibility that consumer preferences exhibit a home bias) reinforces our results: home bias tends to reduced cross-hauling. But the reduction in cross-hauling under competition is smaller than the reduction that a profit maximizing cartel would choose, which in turn is smaller than the reduction that would be consistent with welfare maximization.

An important qualification should be made at this point. In the standard Hotelling framework, as long as each consumer's surplus (gross of the price she pays and the disutility from consuming a variety other than her ideal) is sufficiently high, there is no deadweight loss (DWL) from monopoly. Any DWL of monopoly would come from a downward-sloping aggregate demand curve, which is assumed away (in the relevant interval). Similarly, our paper abstracts away from aggregate demand effects to focus on whether a shift in the mode of competition from non-cooperative to cooperative, in the presence of costly cross-hauling, can be welfare-enhancing (as argued not least

[9] More broadly, as this discussion suggests, a cartel has the potential to increase total welfare when oligopolistic competition between asymmetric firms leads to welfare-reducing distortions in the allocation of consumers to firms. A collusive arrangement has the potential to reduce these distortions, even when there are no side-payments between the firms. In this particular model, though there are asymmetries between the firms in each market, the firms are symmetric over both markets. This allows us to consider symmetric collusive agreements, which greatly simplifies the analysis. We leave the analysis of the welfare effects of single-market asymmetric cartels, on which there already exists some literature (e.g., Harrington 1991, and Athey and Bagwell 2001), for subsequent research.

by certain industries themselves—see note 5)[10]. It should be clear, however, that welfare gains from mitigating the so-called "destructive competition" need to be balanced against any DWL from monopoly. It is in those industries where volume effects are sufficiently small that our result may hold more relevance. Near-zero aggregate volume effects do not seem to be an unreasonable assumption in industries such as cement and sugar. In mature economies, such an assumption does not seem unreasonable for many household appliances, e.g., refrigerators, ovens, dishwashers, etc. Clearly, for industries in which aggregate demand is sufficiently elastic, our welfare conclusions will be overturned.[11]

To the best of our knowledge, this paper provides a first model where a perfect cartel (i.e. the monopoly outcome) cross-hauls too much product between geographic markets, rather than fully dividing them. Unlike models such as that in Bond and Syropoulos (2008), we do not require constrained cartels—i.e. deviation incentives—for cross-hauling to obtain. In addition to the aforementioned empirical work on spatial cartels, our paper can be related to other strains in the literature. Recent work (Lommerud and Sørgard 2001, Schröder 2007, Bond and Syropoulos 2008) examines the stability of multimarket collusion in the wake of trade liberalization (modeled as a reduction in the trade cost)[12]. Similarly, Davidson (1984), Rotemberg and Saloner (1989) and Fung (1992) study single-market cartel stability in light of unilateral trade policy (e.g. tariffs). By contrast, as noted in footnote 5, we abstract away from incentive constraints, implicitly assuming that firms are sufficiently patient so that the fully collusive outcome—i.e. the monopoly cartel—is feasible. An old literature on spatial ("basing-point") pricing and "quasi-cooperation" dates back at least to Smithies (1942). Needham (1964) argues that in the absence of side-payments cross-hauling may actually arise under collusion in order to stabilize a cartel. More recently, Thisse and Vives (1988) investigate the profitability of different pricing schemes in spatial markets and conclude that mill-pricing with full pass-through of transport costs could yield higher profits than

[10] A similar line of "defense" is often heard in merger cases, where merger proponents argue that any unilateral effects will be outweighed by cost efficiencies. We thank Joe Farrell for pointing this out. (In this vein, Banal-Estanol 2007 argues that mergers can improve efficiency through information sharing.)

[11] In differentiated markets, aggregate demand is a function of both prices, and hence its elasticity is not uniquely defined. An alternative specification to ours would be to embed a downward-sloping demand at each point of the Hotelling interval (consumer type). By changing the elasticity of these embedded demand curves, we could change the aggregate demand elasticity while holding the degree of product differentiation constant. We believe that this more general specification would add notational clutter without providing additional intuition beyond what is already given here.

[12] The concern in this strain of the literature is that "cartels are bad" and so if trade liberalization helps to stabilize a cartel, then we should worry about trade liberalization. By contrast, our point is that in certain industries "cartels can be good." Moreover, the comparative statics with respect to trade costs do not have a qualitative impact on the results of our paper as long as trade is positive, though they certainly do affect their magnitudes.

charging the same price in all markets, even though the latter is always the equilibrium in their modeling framework.[13] In our framework, mill-pricing is socially optimal, but the optimal cartel price is intermediate to that of either of the two pricing extremes. However, in a result that is somewhat reminiscent of those in Thisse and Vives (1988), a cartel would choose a price that is closer to mill-pricing relative to the price that would result under competition.[14]

In the trade literature, Brander and Krugman (1983) show that exogenously moving from autarky (there is, by definition, no cross-hauling) to trade competition in a homogeneous-good Cournot oligopoly where aggregate demand slopes downward—and with free entry—is welfare-enhancing: "(t)he pro-competitive effect of having more firms and a larger overall market dominates the loss due to transport costs in this second best imperfectly competitive world" (p.314). With entry barriers, "wasteful" two-way trade in the identical good might lower welfare, when the trade cost is high enough. Indeed, in an extension section of the paper that studies autarky, we obtain a similar result that autarky welfare-dominates market-based "trade regimes" (collusion and competition) except when the trade cost is low[15]. Friberg and Ganslandt (2008) extend Brander and Krugman's (1983) welfare analysis of autarky (the no-entry case) to a linear-demand differentiated-goods Bertrand oligopoly. When market structure is sufficiently concentrated, they find that trade competition is welfare-enhancing relative to autarky[16].

2 The basic model

We consider a geographically segmented industry where goods are horizontally differentiated. To capture the geographic component, we model two local markets, 1 and 2. Shipping product from one market to another—cross-hauling—incurs a unit trade cost $t > 0$. To capture the taste component, we model each local market as a continuum of consumers distributed uniformly over a

[13] Thisse and Vives (1988) consider only three different pricing schemes: (i) the same price in all markets, (ii) mill pricing plus transport-cost pass through (which they refer to as uniform pricing), and (iii) mill pricing plus pseudo-transport costs from some common to the two firms location. In our model, we do not restrict ourselves to this set of pricing rules, but unlike in Thisse and Vives (1988) there are only two distinct locations.

[14] In a different setting, Foros et al. (2002) analyze a type of semi-collusion where firms may collude at the investment stage (where there exist spillover effects), but always compete in the product market. Since collusion at the investment stage allows the positive external effects to be internalized (under collusion, *all* benefits from the investment are credited to the investing firm), they find that it is welfare-enhancing to let firms semi-collude whenever spillover effects turn out to be sufficiently strong.

[15] Other studies where trade lowers welfare, albeit in very different contexts, include Newbery and Stiglitz (1984) and Eden (2007).

[16] Unlike us, neither Brander and Krugman (1983) nor Friberg and Ganslandt (2008) (nor, similarly, Clarke and Collier 2003) are concerned with collusion (i.e. its effect on the magnitude of cross-hauling and welfare).

unidimensional space of product characteristics, defined by the interval $[0,1]$. The disutility from consuming a variety other than one's ideal variety is linear in the distance along this Hotelling interval, with slope $\theta > 0$. There are two firms, A and B, each firm producing one variety. In geographic space, firm A's plant is located in market 1 while firm B's plant is located in market 2. In product space, firm A's product is located at the left endpoint of the unit interval while firm B's product is located at the right endpoint. The two firms have the same constant marginal cost of production $c > 0$.

Consumers make discrete choices, purchasing one unit or none. Let $x \in [0,1]$—the consumer's type—denote the distance from the left endpoint of the unit interval. Consider either one of the local markets, and denote the vector of prices by $p = (p_A, p_B)$; prices can vary across the two markets (though not across consumers) but, for simplicity, we momentarily omit market subscripts. Denoting the reservation price for one's ideal product (relative to the outside good) by V, consumer type x's (ordinary) demand for good A is

$$q_A(p;x) = \begin{cases} 1 & \text{if} \quad U_A(p_A;x) := V - \theta x - p_A \geq V - \theta(1-x) - p_B \text{ and } U_A(p_A;x) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

while her demand for good B is

$$q_B(p;x) = \begin{cases} 1 & \text{if} \quad U_B(p_B;x) := V - \theta(1-x) - p_B > V - \theta x - p_A \text{ and } U_B(p_B;x) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(Notice that by specifying a common V across markets, we do not assume any home bias, but this introduced below by specifying $V_{1A} > V_{1B}$.) The location of the "marginal consumer" $\tilde{x}$, defined as the consumer who is indifferent between goods A and B, follows from solving $U_A(p_A;\tilde{x}) = U_B(p_B;\tilde{x})$, i.e.

$$\tilde{x}(p) = \frac{\theta - p_A + p_B}{2\theta} \tag{1}$$

where $p = (p_A, p_B)$.

We next consider equilibrium outcomes under alternative competitive regimes, price competition and full collusion. To focus on the case of interest, we restrict the space of parameters as follows (see below for verification):

Assumption A1 ("cross-hauling under collusion") $t < 2\theta$: Restricting the cost of cross-hauling between markets to be sufficiently low (relative to the degree of product differentiation) implies that cross-hauling occurs even in the fully collusive regime[17].

Assumption A2 ("full market coverage") $2V \geq 3\theta + 2c + t$: Restricting the reservation price for one's ideal product to be sufficiently high (relative to the degree of product differentiation and the cost primitives) implies that even in the competitive regime the consumer located at $\tilde{x}$, who is indifferent between the inside goods A and B, prefers these to the outside good[18].

Quantity shares for firms A and B are then given by $s_A(p) = \tilde{x}(p)$ and $s_B(p) = 1 - \tilde{x}(p)$ respectively.

2.1 Price competition

Since marginal cost is flat in output, the problem is separable (and analogous) across the two local markets. We consider market 1. In a competitive equilibrium in prices, prices solve the system

$$\begin{cases} \max_{p_A} (p_A - c)\, s_A(p) \\ \max_{p_B} (p_B - c - t)\, s_B(p) \end{cases}$$

with first-order conditions (FOCs)

$$(\theta - p_A + p_B) / (2\theta) - (p_A - c) / (2\theta) = 0$$

$$(\theta + p_A - p_B) / (2\theta) - (p_B - c - t) / (2\theta) = 0$$

[17] In the competitive regime, cross-hauling occurs if $t < 3\theta$. We subsequently discuss the case $2\theta \leq t < 3\theta$, where cross-hauling occurs in the competitive regime but not in the collusive one.

[18] The restriction is equivalent to $\left(U_B(p_B^C; \tilde{x}^C) =\right) U_A(p_A^C; \tilde{x}^C) \geq 0$ (where C denotes the competitive outcome). Say that parameters are such that A2 binds. Now consider a small positive shock to the marginal cost of production, $\Delta c > 0$. As we subsequently show, in the competitive regime costs pass through to prices, so consumers located in the vicinity of $\tilde{x}\left(p^C\right)$ would now buy the outside good: the market would no longer be fully covered (and one can show that a proportion $2\Delta c/\theta$ would no longer be served). By contrast, in the fully collusive (joint maximizing, denoted by JM) regime, the cost shock would not pass through to prices but would be absorbed by the cartel's margins. This regime's marginal consumer (located at $\tilde{x}\left(p^{JM}\right)$, has zero surplus and whom it is profitable to serve, as we later show) would carry on buying (an) inside good.

yielding prices

$$p_{1A}^C = \theta + c + \frac{1}{3}t, \quad p_{1B}^C = \theta + c + \frac{2}{3}t \tag{2}$$

and profits

$$\Pi_{1A}^C = \frac{1}{18\theta}(3\theta + t)^2, \quad \Pi_{1B}^C = \frac{1}{18\theta}(3\theta - t)^2 \tag{3}$$

(now adding market subscripts, and where the superscript C denotes the competitive equilibrium). In equilibrium, the location of the marginal consumer, and thus the quantity share of home firm A, is given by

$$\tilde{x}_1^C = \frac{1}{6\theta}(3\theta + t) = \frac{1}{2} + \frac{1}{6}\frac{t}{\theta}$$

Notice that A1 implies that $\tilde{x}_1^C < 1$ and cross-hauling occurs. (It is clear that we have an interior solution for $\tilde{x}_1^C$ as long as $t < 3\theta$.) Equilibrium outcomes in market 2 are obtained from interchanging the market-firm subscripts (and $\tilde{x}_2^C = 1 - \tilde{x}_1^C$). It is easy to show that a firm's total profit $\Pi_{1A}^C + \Pi_{2A}^C$ is increasing in both the trade cost t and (given A1) the degree of product differentiation θ; intuitively, increasing t or θ relaxes price competition[19].

2.2 Full collusion supported by punishment strategies

We now derive the fully collusive outcome, assuming that this can be sustained by grim trigger strategies that account for the multimarket nature of the firms' contact. These strategies prescribe that in each market each firm sets a fully collusive price, unless the price set by any firm in any one market in the previous period, observed in the current period, differs from the fully collusive price, in which case the prescription is to set the static equilibrium price derived earlier, given (for market 1) by (2). We relegate the specification of the perfect cartel's incentive constraint to the appendix. We simply note here that with firm symmetry across both markets, profit maximizing collusion will in general be supported punishments strategies if the two firms are sufficiently patient, i.e., if their discount factor is sufficiently close to 1.

Clearly, in a fully collusive—or joint-profit maximizing (denoted by the superscript JM)—outcome, prices set by the firms leave the marginal consumer in each market, given by (1), with zero

[19] Proof of this statement follows from noting that $\Pi_{1A}^C + \Pi_{2A}^C = (9\theta^2 + t^2)/(9\theta)$ increases in t and $\partial(\Pi_{1A}^C + \Pi_{2A}^C)/\partial\theta = 1 - (t/(3\theta))^2 >^{A1} 0$.

surplus. To see this, notice that were the marginal consumer to have positive surplus, joint profits could increase by slightly raising prices (and recall that we have assumed that V is large enough that it is profitable to serve all consumers[20]). Thus, fully-collusive prices satisfy $U_A(p_A;\tilde{x}(p)) = 0$ (a condition that, from the definition of $\tilde{x}$, is equivalent to $U_B(p_B;\tilde{x}(p)) = 0$), which from (1) can be rewritten as $2V - \theta - p_A - p_B = 0$. Using this locus of prices, the perfect cartel's problem

$$\max_{p_A,p_B} (p_A - c)\, s_A(p) + (p_B - c - t)\, s_B(p) \quad \text{subject to} \;\; U_A(p_A;\tilde{x}(p)) \geq 0$$

$$\Leftrightarrow \max_{p_A,p_B} (p_A - c)\, \frac{\theta - p_A + p_B}{2\theta} + (p_B - c - t)\left(1 - \frac{\theta - p_A + p_B}{2\theta}\right) \; \text{s.t.} \; U_A(p_A;\tilde{x}(p)) \geq 0$$

collapses to the univariate problem

$$\max_{p_A} (p_A - c)\, \frac{V - p_A}{\theta} + (2V - \theta - p_A - c - t)\left(1 - \frac{V - p_A}{\theta}\right)$$

with FOC

$$\frac{V - p_A}{\theta} - \frac{p_A - c}{\theta} - \frac{\theta - V + p_A}{\theta} + \frac{2V - \theta - p_A - c - t}{\theta} = 0$$

yielding prices

$$p_{1A}^{JM} = V - \frac{1}{2}\theta - \frac{1}{4}t, \;\; p_{1B}^{JM} = 2V - \theta - p_{1A}^{JM} = V - \frac{1}{2}\theta + \frac{1}{4}t \tag{4}$$

and profits

$$\Pi_{1A}^{JM} = \frac{1}{16\theta}\,(2\theta + t)\,(4V - 2\theta - 4c - t)\,, \;\; \Pi_{1B}^{JM} = \frac{1}{16\theta}\,(2\theta - t)\,(4V - 2\theta - 4c - 3t) \tag{5}$$

The location of the marginal consumer is given by

$$\tilde{x}_1^{JM} = \frac{1}{2} + \frac{1}{4}\frac{t}{\theta}$$

[20] We note at this point that at the fully collusive prices derived below, at which the marginal consumer has zero surplus, profit earned on the imported good is positive (though lower than profit on the home good). This follows from A1 and A2, noting that the margin $p_{1B}^{JM} - c - t = V - \frac{1}{2}\theta - c - \frac{3}{4}t = \frac{1}{2}\left(2V - \theta - 2c - \frac{3}{2}t\right) >^{A1} \frac{1}{2}\left(2V - 2\theta - 2c - t\right) > \frac{1}{2}\left(2V - 3\theta - 2c - t\right) \geq^{A2} 0$. As in footnote 18, consider a positive shock to the production cost c. Since the margin on imports in the collusive regime exceeds $\frac{1}{2}\left(2V - 3\theta - 2c - t\right)$, any sufficiently large cost shock that were to make imports barely profitable in the collusive regime would necessarily lead to incomplete market coverage in the competitive regime, since A2 would no longer hold. In other words, incomplete coverage under collusion implies incomplete coverage in the competitive regime, but not the other way round.

where $\tilde{x}_1^{JM} > \tilde{x}_1^C$ such that the quantity share of the home good in the collusive outcome is increased compared to that in the competitive outcome. (Again, interchange the market-firm subscripts for market 2 outcomes, and $\tilde{x}_2^{JM} = 1 - \tilde{x}_1^{JM}$.) It is clear from A1 that, though increased relative to competition, the home firm's share under collusion is less than 1 (we subsequently consider corner solutions[21]). The result is captured in the following proposition.

Proposition 1 In the restricted space of parameters, the joint-profit maximizing outcome involves less cross-hauling than the competitive equilibrium outcome. Despite the home good and the imported good being equally close to the average consumer's ideal variety, the cartel trades less product across geographic markets, or "swaps geographic markets" relative to the competitive regime.

Contrary to the competitive outcome, a firm's total profit $\Pi_{1A}^{JM} + \Pi_{2A}^{JM}$ decreases in both the trade cost t (given A1) and the degree of product differentiation θ. Intuitively, the competitive mechanism is now absent, and a higher t raises the cost of bringing a variety to market, while a higher θ raises the disutility from not consuming one's ideal variety[22] [23].

3 Welfare effects

3.1 Welfare across the competitive and collusive regimes

We compare social welfare—the sum of consumer surplus and producer surplus—across the competitive and fully collusive regimes. Given our focus on the case where the market is fully covered (i.e. in both regimes all consumers purchase one unit of an inside good, with "gross utility" V), we can restrict our comparison of welfare across the two equilibrium outcomes to (i) the different total cost of cross-hauling product between geographic markets, borne by firms, and (ii) the different total disutility ("travel cost") from consuming a variety other than one's ideal, borne by consumers.

[21] Outside A1, for $t \geq 2\theta$, the perfect cartel does not cross-haul: $\tilde{x}_1^{JM} = 1$. From $U_A(p_A; \tilde{x}_1^{JM} = 1) = 0$, the fully collusive price is $p_{1A}^{JM} = V - \theta$ (and $p_{1B}^{JM} = V$), and profit is $V - \theta - c = \frac{1}{2}(2V - 2\theta - 2c) > \frac{1}{2}(2V - 3\theta - 2c - t) >^{\text{A2}} 0$.

[22] Proof of these comparative statics follow from noting that $\partial\left(\Pi_{1A}^{JM} + \Pi_{2A}^{JM}\right)/\partial t = (t - 2\theta)/(4\theta) <^{\text{A1}} 0$ and $\partial\left(\Pi_{1A}^{JM} + \Pi_{2A}^{JM}\right)/\partial\theta = -\left(t^2 + 4\theta^2\right)/\left(8\theta^2\right) < 0$.

[23] Since $\partial\left(\Pi_{1A}^{JM} + \Pi_{2A}^{JM}\right)/\partial t < 0$ and (as noted earlier) $\partial\left(\Pi_{1A}^{C} + \Pi_{2A}^{C}\right)/\partial t > 0$, the (positive) private gains from collusion decrease in the trade cost t: $\partial\left(\Pi_{1A}^{JM} + \Pi_{2A}^{JM} - \Pi_{1A}^{C} - \Pi_{2A}^{C}\right)/\partial t = \partial\left(\frac{1}{2}\left(2V - 3\theta - 2c - t + \frac{1}{36\theta}t^2\right)\right)/\partial t = \frac{1}{36\theta}(t - 18\theta) <^{\text{A1}} 0$. On first thought this may seem counter-intutitive, since the cartel cross-hauls less $(1 - \tilde{x}_1^{JM} < 1 - \tilde{x}_1^C)$. Recall, however, that raising t softens price competition in the competitive regime.

Clearly, the total cost of cross-hauling (into market 1, with market 2 being analogous) under price competition, $t\left(1-\tilde{x}_1^C\right)$, exceeds the cost from cross-hauling under full collusion, $t\left(1-\tilde{x}_1^{JM}\right)$, as there is more trade in the competitive regime.

The total consumer taste disutility under price competition is $\int_0^{\tilde{x}_1^C}\theta x dx+\int_{\tilde{x}_1^C}^1\theta\left(1-x\right)dx=\frac{1}{2}\theta\left(\tilde{x}_1^C\right)^2+\frac{1}{2}\theta\left(1-\tilde{x}_1^C\right)^2=\frac{1}{2}\theta\left(2\left(\tilde{x}_1^C\right)^2-2\tilde{x}_1^C+1\right)$, while that under full collusion, obtained similarly, is $\frac{1}{2}\theta\left(2\left(\tilde{x}_1^{JM}\right)^2-2\tilde{x}_1^{JM}+1\right)$. It should also be clear that the former is lower than the latter, as the marginal consumer in the competitive regime lies closer to the midpoint of Hotelling's unit interval than the marginal consumer in the collusive regime ($\frac{1}{2}<\tilde{x}_1^C<\tilde{x}_1^{JM}$); mechanically, the quadratic expression in brackets defines a convex parabola with minimum at $x=\frac{1}{2}$. Relative to the collusive regime, the good each consumer chooses in the competitive regime is (weakly) closer to her ideal variety.

We now combine the total cost of cross-hauling and the total consumer taste disutility under each of the two regimes to obtain the following result:

Proposition 2 In the restricted space of parameters, social welfare under full collusion exceeds social welfare under price competition, since (i) the (collusive) effect of "swapping"—i.e. reducing cross-hauling between—geographic markets, dominates (ii) the (competitive) effect of consumers choosing products that are closer to their ideal variety. Further, the welfare gains under collusion relative to competition are increasing in the unit trade cost t and decreasing in the degree of product differentiation θ.

Proof From the above analysis, social welfare under full collusion exceeds social welfare under price competition, $W^{JM}>W^C$, if and only if

$$t\left(1-\tilde{x}_1^{JM}\right)+\frac{1}{2}\theta\left(2\left(\tilde{x}_1^{JM}\right)^2-2\tilde{x}_1^{JM}+1\right)<t\left(1-\tilde{x}_1^C\right)+\frac{1}{2}\theta\left(2\left(\tilde{x}_1^C\right)^2-2\tilde{x}_1^C+1\right)$$
$$\Leftrightarrow \frac{7}{144}\frac{t^2}{\theta}>0$$

which holds given $t>0$. The relative welfare gains under collusion, per market, are given by the left-hand-side of the final inequality. ■

It is immediate from (2) and (4) above that there is price discrimination in favor of the imported good (or, equivalently, against the home good) in both the competitive and the collusive regimes. It turns out in this model that price discrimination is *more* pronounced under competition than under collusion, in the sense that $p_{1B}^{C} - p_{1A}^{C} = \frac{1}{3}t < p_{1B}^{JM} - p_{1A}^{JM} = \frac{1}{2}t < t$. (This result is somewhat unusual, as price discrimination is traditionally associated with the exercise of market power[24].) From the social point of view, the competitive equilibrium is characterized by "excessive consumption of the imported variety", or "excessive trade". Collusion serves as a mechanism to correct this failure, but only partially, as we show in the subsequent subsection.

It is also worth pointing out that the welfare gain from collusion occurs even for a low, but positive, unit trade cost. As the trade cost approaches zero, social welfare converges under the two alternative regimes, i.e. as $t \to 0$, we have that $\tilde{x}_1^{JM} \to \tilde{x}_1^{C} \to \frac{1}{2}$. The setup then approaches the standard Hotelling model, in which collusion does not result in welfare losses, leading only to a transfer from consumers to producers.

Quite striking is the further finding that, relative to competition, collusion can be good even for *consumers*, a result we state in the following proposition.

Proposition 3 In the restricted space of parameters, consumer surplus under full collusion may exceed consumer surplus under price competition. In particular, collusion can bring gains to consumers relative to competition when the unit trade cost t is sufficiently high (relative to the reservation price for one's ideal product V).

Proof We compute $CS^{JM} = \int_0^{\tilde{x}_1^{JM}} \left(V - \theta x - p_{1A}^{JM}\right) dx + \int_{\tilde{x}_1^{JM}}^{1} \left(V - \theta\left(1 - x\right) - p_{1B}^{JM}\right) dx$ and, similarly, CS^{C} (we omit expressions for brevity; notice that CS^{C} is equivalent to the area of two trapezia, while CS^{JM} collapses to the area of two triangles as the marginal consumer has zero surplus). Then consider the condition $CS^{JM} - CS^{C} > 0 \Leftrightarrow 5t^2/\left(72\theta\right) - \left(2V - 3\theta - 2c - t\right) > 0$. From A2, $2V - 3\theta - 2c - t$ is positive (and decreasing in t). The first term of the inequality is also positive (and increasing in t). Clearly, when t is high enough and A2 is not too slack, the condition can hold; more formally, $CS^{JM} > CS^{C}$ iff A2 is within $5t^2/\left(72\theta\right)$ of binding. ■

Intuitively, when t is high enough, trade in the competitive regime is so wasteful, and the pass-through of this waste to prices is so high, that collusion is welcomed even by consumers (despite the cartel catering less to the heterogeneity in tastes). When collusion raises consumer welfare relative to competition, the price on the imported good rises, but the price on the home good

[24] Under competition, there is more "dumping" (in the Brander and Krugman 1983 sense) or, borrowing other terms from the trade and spatial literatures, there is a greater degree of "pricing to market" or "freight absorption" (again in the sense that the imported good's price upcharge falls short of the trade cost).

declines.[25] In this region of parameters, consumers who under competition were already buying the home good (and even some near-marginal consumers who were buying their preferred imported good but now switch to the even cheaper home good) are made better off through collusion. This gain in consumer welfare dominates both the loss suffered by consumers who carry on buying the now dearer imported good and the loss experienced by some consumers who have been induced to switch to their less-favored home variety.[26]

3.2 First-best social outcomes

The immediate question then is how distortionary are the market-based behavioral regimes derived above? We now compute the set of first-best outcomes, where social welfare is maximal, and compare them to the competitive and collusive outcomes. As we explain, what characterizes a first-best social outcome is the price *difference* between the home good and the imported good, which is equal to the trade cost. We then provide price levels for two alternative (and extreme) first-best outcomes, where the division of surplus between producers and consumers is reversed: prices set by a "business-friendly" social planner, and prices set by a "consumer-friendly" social planner. To be clear, our planner's bias between pro-business and pro-consumer does not affect the price of the imported good *relative* to the home good, which determines the welfare trade-off between meeting consumers' love of variety and saving on trade costs.

Denote this first-best outcome by the superscript FB and consider market 1 (again market 2 is analogous). Express the location of consumer $\tilde{x}_1^{FB}$, who is indifferent between the two inside goods, as lying at a distance d to the right of the midpoint of the unit interval of product characteristics, i.e. $\tilde{x}_1^{FB} = \frac{1}{2} + d$. For this marginal consumer to be indifferent to buying the home good or the imported good, the fact that she finds the home good less appealing must be offset by a price difference in its favor. The relative taste disutility of the home good is that of traveling a distance $2d$ (a distance d to the midpoint $\frac{1}{2}$, and then another d), costing the marginal consumer $2d\theta$. Now,

[25] To see this, note that $p_{1A}^{JM} < p_{1A}^{C} \Leftrightarrow t/6- \ (2V - 3\theta - 2c - t) > 0$ and $t/6 >^{A1} 5t^2/\,(72\theta)$. So when A2 is within $5t^2/\,(72\theta)$ of binding (and thus $CS^{JM} > CS^{C}$ as per the proof of Proposition 3), A2 is also within $t/6$ of binding and hence $p_{1A}^{JM} < p_{1A}^{C}$. A similar argument shows that the collusive price on the imported good does not fall below the competitive price (intuitively, price discrimination in favor of the imported good is higher in the competitive regime).

[26] Relative to competition, collusion impacts consumer welfare both by shifting prices (up for the imported variety and—for t high—down for the home variety) and by changing the market allocation (against imports). Since the latter effect hurts consumers and is not accounted for in typical calculations of "customer damages," we note that whenever collusion turns out to raise consumer welfare, it is also the case that customer damages will be negative (i.e. no "damage").

the social planner equates this relative disutility $2d\theta$ with the cost of cross-hauling t, i.e. $2d\theta = t$, from which $d = t/(2\theta)$ and the location of the marginal consumer follows:

$$\tilde{x}_1^{FB} = \begin{cases} \frac{1}{2} + \frac{1}{2}\frac{t}{\theta} & \text{if } t < \theta \\ 1 & \text{otherwise} \end{cases}$$

Relative to the trade-prone competitive and collusive regimes, the social planner reduces wasteful cross-hauling across borders, opting for less trade—and none at all when $\theta < t < 2\theta$— and a greater quantity share for the home good: $1 \geq \tilde{x}_1^{FB} > \tilde{x}_1^{JM} > \tilde{x}_1^{C}$. There is no price discrimination against the home good, as the price difference is equated to the trade cost t, in contrast to the market-based regimes where price discrimination was substantial (i.e. the price difference was as low as $\frac{1}{2}t$ under collusion and $\frac{1}{3}t$ under competition).

We summarize the welfare result in the following proposition.

Proposition 4 In the restricted space of parameters, social welfare under full collusion—though higher than under price competition—is suboptimal. Relative to the fully collusive outcome, a social planner would raise the price of the imported good *relative* to the price of the home good, further restricting the penetration of imports, i.e. the social planner would further enhance geographic market-swapping at the expense of consumers' taste for variety.

Proof When $0 < t < \theta$, the (per market) increase in social welfare in a first-best outcome relative to full collusion is

$$\begin{aligned} W^{FB} - W^{JM} &= t\left(1 - \tilde{x}_1^{JM}\right) + \frac{1}{2}\theta\left(2\left(\tilde{x}_1^{JM}\right)^2 - 2\tilde{x}_1^{JM} + 1\right) - t\left(1 - \tilde{x}_1^{FB}\right) - \frac{1}{2}\theta\left(2\left(\tilde{x}_1^{FB}\right)^2 - 2\tilde{x}_1^{FB} + 1\right) \\ &= \frac{1}{16}\frac{t^2}{\theta} > 0 \end{aligned}$$

This is increasing in t and decreasing in θ. When $\theta < t < 2\theta$,

$$W^{FB} - W^{JM} = \frac{1}{16\theta}(3t - 2\theta)(2\theta - t) > 0$$

Notice that $\partial\left(W^{FB} - W^{JM}\right)/\partial t = (4\theta - 3t)/(8\theta)$ which is positive for low t and negative for high t. Also, $\partial\left(W^{FB} - W^{JM}\right)/\partial\theta = \left(3t^2 - 4\theta^2\right)/\left(16\theta^2\right)$ which is negative for low t and positive for high t. ■

Intuitively, though an improvement over competition, the perfect cartel still price discriminates against the home good and still imports too much product, as it is eager to cater to consumers' taste for variety in order to extract maximal surplus. Seen from the socially optimal outcome, the cartel disproportionately raises the price of the home good, which has a large market share, leading to large revenue gains from the sales to (the many) inframarginal consumers. As a result, the cartel does not fully internalize the social cost of cross-hauling product. By contrast, the social planner cures this externality.

Now, to illustrate, consider alternative price levels within the set of first-best social outcomes. A pro-business ("$pro-b$") social planner, wishing to maximize producer surplus conditional on total welfare being optimal, would set prices such that the marginal consumer's surplus is fully extracted, $U_A(p_A;\tilde{x}_1^{FB})=0$, that is

$$p_{1A}^{FB,pro-b}=V-\theta\tilde{x}_1^{FB}=V-\frac{1}{2}\min\left(\theta+t,2\theta\right),\quad p_{1B}^{FB,pro-b}=p_{1A}^{FB,pro-b}+t=V+t-\frac{1}{2}\min\left(\theta+t,2\theta\right)$$

Profits are then given by

$$\Pi_{1A}^{FB,pro-b}=\begin{cases}\frac{1}{4\theta}\left(\theta+t\right)\left(2V-\theta-2c-t\right) & \text{if } t<\theta\\ V-\theta-c & \text{otherwise}\end{cases}$$

$$\Pi_{1B}^{FB,pro-b}=\begin{cases}\frac{1}{4\theta}\left(\theta-t\right)\left(2V-\theta-2c-t\right) & \text{if } t<\theta\\ 0 & \text{otherwise}\end{cases}$$

whereby, as in the collusive regime, a firm's total profit in this business-friendly first-best solution $\Pi_{1A}^{FB,pro-b}+\Pi_{2A}^{FB,pro-b}$ decreases in both the trade cost t (until t reaches θ) and the degree of product differentiation θ. Further, total profit in this solution falls short relative to the fully collusive outcome[27]:

$$\Pi_{1A}^{FB,pro-b}+\Pi_{2A}^{FB,pro-b}-\left(\Pi_{1A}^{JM}+\Pi_{2A}^{JM}\right)=\begin{cases}-\frac{1}{8}\frac{t^2}{\theta}<0 & \text{if } t<\theta\\ -\frac{1}{8\theta}\left(2\theta-t\right)^2<0 & \text{otherwise}\end{cases}$$

Alternatively, a pro-consumer ("$pro-c$") social planner, wishing to maximize consumer surplus conditional on total welfare being optimal, would set prices such that firms break even[28]:

$$p_{1A}^{FB,pro-c}=c,\quad p_{1B}^{FB,pro-c}=p_{1A}^{FB,pro-c}+t=c+t$$

Much as we compared firm profits in the preceding business-friendly first-best solution relative to the collusive outcome, we may wish to compare consumer surplus in this consumer-friendly first-best solution $CS^{FB,pro-c}$ relative to the competitive outcome CS^C. Computing $CS^{FB,pro-c}=\int_0^{\tilde{x}_1^{FB}}\left(V-\theta x-p_{1A}^{FB,pro-c}\right)dx+\int_{\tilde{x}_1^{FB}}^1\left(V-\theta\left(1-x\right)-p_{1B}^{FB,pro-c}\right)dx$ and, similarly, CS^C, we then take the difference to obtain

$$CS^{FB,pro-c}-CS^C=\begin{cases}\frac{1}{9\theta}\left(9\theta^2+2t^2\right) & \text{if } t<\theta\\ \frac{1}{36\theta}\left(27\theta^2+t\left(18\theta-t\right)\right) & \text{otherwise}\end{cases}$$

from which it follows that consumer surplus rises relative to the competitive regime, i.e. $CS^{FB,pro-c}>CS^C$ (recall A1)[29].

Figure 1 summarizes quantity shares and (relative) prices in market 1 in each of the three

[27] Note that since the price of the imported good in this first-best outcome rises relative to that in the fully collusive equilibrium, the price-cost margin remains positive. Since there is no price discrimination, price-cost margins on the home and imported goods are now equal (unlike in the collusive and competitive regimes where the home good had higher margins relative to the imported one).

[28] Note that prices for both the home good and the imported good fall relative to the competitive outcome. To see this, note that $p_{1A}^{FB,pro-c}-p_{1A}^C=-\theta-\frac{1}{3}t<0$ and $p_{1B}^{FB,pro-c}=c+t<\frac{1}{2}t+c+\frac{2}{3}t<^{\text{A1}}\theta+c+\frac{2}{3}t=p_{1B}^C$. Unsurprisingly (see below), consumer surplus in the consumer-friendly first-best solution increases relative to the competitive outcome.

[29] Conceptually, this consumer-friendly first-best solution should not be confused with the optimization of consumer surplus, as it maximizes the sum of consumer and producer welfare. It is easy to show, however, that in this case both solutions coincide. The argument is based on the observation that producer surplus under the consumer-friendly first-best solution is zero. Thus, the entire surplus is obtained by the consumers. Since total surplus is maximal and the individual rationality constraints must be satisfied, there is no possible allocation that would yield higher surplus to consumers.

regimes. (The bottom panel is drawn for $t < \theta$ such that the social planner would allow imports to penetrate.) Under all three regimes, the quantity share of the home good increases in the trade cost t and decreases in the degree of product differentiation θ. In particular, as $t \to 0$, the location of the marginal consumer in all three regimes approaches the midpoint of the unit interval and each consumer, facing common prices across goods, buys the good that more closely resembles her ideal product; as mentioned above, we are now in the standard Hotelling setup.

Moving in the other direction, consider for a moment raising the trade cost beyond the upper bound set by A1, $t < 2\theta$. Figure 2 summarizes the quantity cross-hauled (and welfare), under each regime, on moving beyond the restricted space of parameters, as a function of t. As noted, for $t \geq 2\theta$, the fully collusive cartel would now make the same trade-off as would the social planner between meeting consumers' taste for variety and saving on trade costs, which would be to not cross-haul at all, i.e. in this region, $\tilde{x}_1^{JM} = \tilde{x}_1^{FB} = 1$. Social welfare under full collusion would then be maximal, i.e. $W^{JM}\big|_{t \geq 2\theta} = W^{FB}$. Specifically, for $2\theta \leq t < 3\theta$, while collusion would eliminate cross-hauling, this would still not be the case under price competition[30]. Further raising t, for $t \geq 3\theta$, cross-hauling would now cease also in the competitive regime, $\tilde{x}_1^{C} (= \tilde{x}_1^{JM} = \tilde{x}_1^{FB}) = 1$, with competition now also yielding optimal social welfare[31]. Notice the "concavity" of the problem: at the low end, as $t \to 0$, all three regimes coincide in terms of the degree of geographic market-swapping—this is minimal—and welfare—the consumer taste disutility is minimal, as each consumer acquires the good that is closest to her ideal. Again, this is standard Hotelling. At the high end, as t reaches 3θ, all three regimes again coincide: the degree of market-swapping is now maximal—there is no cross-hauling—and both competition and collusion are first-best.

Before proceding to approaches to implement the first-best, we note that the inefficiency of oligopolistic competition in markets with asymmetric firms is a common finding in the literature. Its source in our paper is the price discrimination against consumers that a firm has the comparative cost advantage in serving.[32]

[30] There would thus still be welfare gains from collusion relative to competition: similar to the earlier welfare calculations, noting that $\tilde{x}_1^C < \tilde{x}_1^{JM} = 1$, we have $W^{JM} - W^C = t\left(1 - \tilde{x}_1^C\right) + \frac{1}{2}\theta\left(2\left(\tilde{x}_1^C\right)^2 - 2\tilde{x}_1^C + 1\right) - \frac{1}{2}\theta = (5t - 3\theta)(3\theta - t)/(36\theta) > 0$ for $2\theta \leq t < 3\theta$.

[31] It is easy to show that at $t = 3\theta$ assumption A2 ("full coverage") binds.

[32] See Bester and Petrakis (1996) for a model in which price discrimination in a Hotelling market reduces efficiency and reduces firm profits, and the generalization in Liu and Serfes (2004). Our framework goes beyond this literature in examing the effects of collusion relative to both competition and the first best, as well as examining the impact of physical transportation costs.

3.3 Government interventions: Tax and subsidy policies, and price regulation

We show that a social planner, rather than setting prices directly (as suggested in Proposition 3), can replicate the socially optimal market allocation through a system of taxes and subsidies. Say that a government, with oversight responsibility over the two local markets, can (in each market) impose a unit tax (tariff) $\tau \geq 0$ on sales of the imported good and a unit subsidy $\omega \geq 0$ on sales of the home good. The tax and subsidy policy is set prior to the firms setting prices. (Alternatively, in an international context, one can envision two countries coordinating to reciprocally tax imports and subsidize sales of the domestically-produced variety.) For a market-based regime with either price competition or full collusion, the following proposition describes the symmetric tax and subsidy policy that yields the welfare optimal market allocation.

Proposition 5 In the restricted space of parameters, an appropriate tax and subsidy policy can be used to induce the market-based regime—either price competition or full collusion—to limit trade across geographic markets to the socially optimal level. In particular, an optimal unit tax on imports and unit subsidy on home good sales pair (τ, ω) satisfies (i) $\tau + \omega = 2t$ in the competitive regime, and (ii) $\tau + \omega = t$ in the collusive regime.

Proof See the appendix. ■

The proposition states necessary conditions for the first-best market allocation, $\tilde{x}_1^{FB} \left(> \tilde{x}_1^{JM} > \tilde{x}_1^{C}\right)$, to be replicated in both the competitive and collusive regimes. The reason why these conditions are not sufficient is that individual rationality constraints, for both firms and consumers, need to be satisfied as well. Consider an example, for each regime, of a policy that attains first-best. In the competitive regime, the social planner could optimally tax the imported good at $\tau = 2t/3$ and subsidize the home good at $\omega = 4t/3$. In the collusive regime, the social planner could optimally tax the imported good at $\tau = t$ and not subsidize the home good. (It turns out that the reason why only a tax on the imported good does not suffice in the competitive regime is that in the space of parameters we consider, the market would not always be fully covered. See the appendix.)

Intuitively, as can be seen in the appendix, an optimal tax and subsidy would induce competitive or cartelized firms to set prices such that the price of the imported good exceeds the price of the home good exactly by t. That is, price discrimination against purchases of the home good is eliminated, and excessive cross-hauling ceases.

A natural question concerns how government, facing the "highly wasteful" competitive regime, can implement a (suboptimal) tax and subsidy policy to replicate the market allocation observed in the "less wasteful" collusive regime $\tilde{x}_1^{JM}\left(> \tilde{x}_1^C\right)$ (i.e. as calculated in Section 2.2, free of tax and subsidy). As we show in the appendix, a necessary (though again insufficient) condition for government to replicate the cartel's market allocation is that the (τ, ω) pair satisfies $\tau + \omega = \frac{1}{2}t$. For example, the competitive duopoly would be induced to cross-haul the same amount of product as the cartel would (or, equivalently, price discriminate just as the cartel would, $p_{1B}^{JM} - p_{1A}^{JM} = \frac{1}{2}t$) were, say, $(\tau, \omega) = (\frac{1}{4}t, \frac{1}{4}t)$. In this particular example, as we show, both consumer surplus and firm profits turn out to be higher relative to the tax-and-subsidy-free competitive outcome.

In sum, tax and subsidy policies can be used in any regime to replicate another regime's market allocation scheme, namely (and intuitively, recalling Figure 1) (i) $\tilde{x}_1^C \xrightarrow{\tau+\omega=2t} \tilde{x}_1^{FB}$, (ii) $\tilde{x}_1^{JM} \xrightarrow{\tau+\omega=t} \tilde{x}_1^{FB}$ and (iii) $\tilde{x}_1^C \xrightarrow{\tau+\omega=\frac{1}{2}t} \tilde{x}_1^{JM}$, with the latter characterizing an "intermediate" (suboptimal) level of intervention.

As an alternative to this tax and subsidy policies, the government can implement the first best through price regulation. Though a direct command-and-control price regulation is not likely to be feasible, the first best can be achieved through much simpler (and politically feasible) regulation. The key insight on how to accomplish this is that observe that in our model, in which aggregate effects are absent, inefficiency arises solely from the fact that an importer chooses to absorb a portion of the freight costs. In other words, a firm price discriminates against the consumers in its own territory. If the governement prohibits such price discrimination, the first best allocation of consumers to firms will ensue. This can be accomplished by simply forcing the firms to charge a single "mill price" for both markets. The firms would either pass through the transport (or shipping costs) to the consumers (this would require that the government know the level of these costs so as to enforce the regulation) or it would be required that transportation be done by a competitive third-party industry that would (in equilbrium) charge a price equal to the transport costs.

3.4 Can autarky improve welfare over market-based trade regimes?

We have examined the extent of cross-hauling—or trade—under two alternative market-based "trade regimes"—competitive trade and collusive trade—comparing this to the socially-optimal amount of trade (recall Figure 2 for a summary). By contrast, the trade literature typically compares one trade regime (competitive trade, say) against the absence of trade, or autarky. For completeness, we now derive the autarkic outcome and study the welfare effect of autarky relative to the trade equilibria (both the competitive and the collusive regimes).

Is it possible that certain (competitive or even collusive) markets lead to so much wasteful trade that society is better off in autarky? Instinctively, one would think not. Imagine the two markets initially shut off from each other. Why would a government forestall the entry in each market of a high-cost (importing) firm that better meets the tastes of some consumers and (recalling that a perfect cartel engages in trade) is privately profitable? Somewhat surprisingly, however, we show that the government can improve welfare over market-based trade regimes by directly imposing autarky. The reason is that once entry is allowed, the government cannot dictate the scale of entry. Though autarky can only be worse than allowing limited trade, under first-best, it can be better than allowing unrestricted trade. In other words, it is ideal if the government can limit trade, say through a tax and subsidy policy, but if that is not possible (say because, in an international setting, only the blunt instrument of blocking entry through health/safety regulations is available, rather than the finer instrument of tariffs), then it may make sense to ban trade outright.

In the autarkic regime (denoted by the superscript $AUTK$), each market is a monopoly. (Due to the symmetry, we again consider market 1, and thus firm A, and omit market-firm subscripts.) Within the space of parameters restricted by A1 and A2, there are two kinds of monopoly outcomes. The first case occurs for V high enough that the autarkic monopolist fully covers the market. As can be verified below, this occurs for $V \geq 2\theta + c$. (In the appendix, we show that a sufficient, though not necessary, condition for $V \geq 2\theta + c$ to hold, given A2, is $t \geq \theta$.) In this full coverage case, the monopolist sets price such that the consumer at $x = 1$ has zero surplus, i.e. $V - \theta - p^{AUTK} = 0$, so $p^{AUTK} = V - \theta$ and thus $\Pi^{AUTK} = V - \theta - c$. In the second (complementary) case, where $V < 2\theta + c$, full coverage is not optimal for the monopolist. At the monopoly price, the consumer at $\check{x}^{AUTK} < 1$ is indifferent between the inside good and the outside good, i.e. $V - \theta\check{x}^{AUTK} - p^{AUTK} = 0$, or $\check{x}^{AUTK} = \left(V - p^{AUTK}\right) / \theta < 1$.[33] The monopolist's problem in this case is then:

$$\max_{p} \left(p - c\right) \frac{V - p}{\theta}$$

yielding $p^{AUTK} = \frac{1}{2}\left(V + c\right)$ and $\Pi^{AUTK} = \left(V - c\right)^2 / \left(4\theta\right)$, and where the share of the inside good

[33] We use $\check{x}$ rather than $\tilde{x}$ since all along $\tilde{x}$ has denoted the location of the consumer who—when trade is allowed—is indifferent between either inside good A or B.

is $\breve{x}^{AUTK} = (V - c) / (2\theta) < 1$. In summary, the autarkic price and per-market profit are given by

$$p^{AUTK} = \begin{cases} \frac{1}{2}(V + c) \quad \text{if } V < 2\theta + c \text{ (incomplete coverage)} \\ V - \theta \quad \text{otherwise (full coverage)} \end{cases}$$

and

$$\Pi^{AUTK} = \begin{cases} \frac{1}{4\theta}(V - c)^2 \quad \text{if } V < 2\theta + c \text{ (incomplete coverage)} \\ V - \theta - c \quad \text{otherwise (full coverage)} \end{cases}$$

The following proposition states regions in parameter space for which society would be better off under autarky relative to market-based trade regimes. Put simply, autarky welfare-dominates market-based trade regimes except when the trade cost t is low; for these low t cases, the welfare shortfall under autarky narrows as t is raised.

Proposition 6 In the restricted space of parameters, for a sufficiently high unit trade cost, social welfare under autarky exceeds social welfare under full collusion (and thus exceeds social welfare under price competition). In particular, $W^{AUTK} > W^{JM} \left(> W^{C}\right)$ holds (i) for $\theta \leq t\,(< 2\theta)$ (here there is full market coverage under autarky); and (ii) for $\frac{2}{3}\theta < t < \theta$ *and* $V \geq 2\theta + c$ (i.e. whenever there is full market coverage under autarky). Further, (iii) for $\frac{3}{5}\theta < t \leq \frac{2}{3}\theta$ *and* $V \geq 2\theta + c$, social welfare under autarky exceeds social welfare under price competition but is lower than social welfare under full collusion, i.e. $W^{JM} \geq W^{AUTK} > W^{C}$. Outside these regions, any welfare shortfall under autarky relative to price competition (and thus relative to full collusion) narrows as the trade cost increases. In particular, (iv) for $t \leq \frac{3}{5}\theta$ *and* $V \geq 2\theta + c$, increasing t raises both $W^{AUTK} - W^{C} \leq 0$ and $W^{AUTK} - W^{JM} < 0$ toward zero; and (v) for $V < 2\theta + c$ (market coverage is incomplete under autarky, occurring for $t \leq 2V - 3\theta - 2c < \theta$), increasing t raises both $W^{AUTK} - W^{C} \lesseqgtr 0$ and $W^{AUTK} - W^{JM} \lesseqgtr 0$ toward zero (and possibly beyond).

Proof See the appendix. ■

Notice that statement (i) of the proposition is quite intuitive. Recall that in the interval $\theta \leq t\,(< 2\theta)$ the first-best social outcome involves no cross-hauling—unlike the collusive regime, let alone the competitive regime, where cross-hauling obtains. Since in this region the autarkic monopolist would fully cover the market, autarky is thus first-best. Statements (ii) through (iv) pertain also to regions where there is full market coverage in autarky: conditional on full coverage, autarky is preferred to price competition (if not to full collusion) for t no less than $\frac{3}{5}\theta$. Statement (v) says that in the region where there is incomplete market coverage in autarky (this is a strict subspace of $t < \theta$), it may be that either $\left(W^{JM} >\right) W^{C} > W^{AUTK}$, $W^{JM} > W^{AUTK} \geq W^{C}$ or $W^{AUTK} \geq W^{JM} \left(> W^{C}\right)$; importantly, however, as t increases in this region (holding other parameters fixed) the undesirability of autarky relative to market-based regimes diminishes and may be reversed.[34]

3.5 The Effects of Home Bias

Very often, consumers in a market favor local products over competing imported products. Part of the reason may be national sentiment, sometimes encapsulated in ad campaigns (witness the frequent prominent labeling of "Made in the U.S.A" in products produced and sold in the Amerinca market), or it could be simply be that national brands tend to evolve to appear to national tastes, or it could be part of a consumer movement motivated by other considerations (e.g., the green movement encourages the purchase of locally produced goods to reduce greenhouse emmissions generated from transportation). What would the effect of "home bias" in consumer preferences be on the cross-hauling under each of the three environments we consider above. It is clear that if consumers favor the local brand, there will be less cross-hauling in under all environments; but it is not clear whether they will be all affected equally. To fix ideas, we introduce home-bias by assuming that consumers have an additional willingness to pay h for the locally produced product. Considering market 1, in which product A is produced, the location of the consumer who is indifferent between purchasing the local product and the import would now be given by:

$$\tilde{x}^{HB}(p) = \frac{\theta - p_A + p_B + h}{2\theta} \tag{6}$$

[34] A comment on unilateral trade policy is in order. Say that a local market's government can ban imports but still allow outbound trade (i.e. exports) and that the other local market does not reciprocate. We know that foreign sales are profitable for the home firm. So whenever autarky is preferred over a trade regime, it must be the case that a unilateral import ban welfare-dominates autarky (i.e. the banning of inbound and outbound trade). Further, whenever autarky lowers welfare relative to (reciprocal) trade, it may be that a unilateral import ban still welfare-dominates trade.

Repeating the derivations in the preceding sections, we find that the equilibrium prices under competition are given by

$$p_{1A}^{C-HB} = \theta + c + \frac{1}{3}t + \frac{1}{3}h, \quad p_{1B}^{C-HB} = \theta + c + \frac{2}{3}t - \frac{1}{3}h \tag{7}$$

and the equilibrium location of the indifferent consumer by

$$\tilde{x}_1^{C-HB} = \frac{1}{6\theta}\left(3\theta + t + h\right) = \frac{1}{2} + \frac{1}{6}\frac{t}{\theta} \tag{8}$$

An increase in home bias reduces the market share of the imported good, as expected. Even though an increase in home bias reduces the equilibrium price of the imported good, this is not sufficient to fully counter-act the increased willingness of domestic consumers to pay for the domestic good, and the share of the domestic good increases.

Now let us turn our attention to market shares under collusion. Recall that the cartel would set prices so that the marginal consumer gets zero surplus (under our assumption that it is profit maximizing to cover the entire market). Its joint profits from market 1 (market 2 would generate equal profits, and the corresponding expression could be obtained by inter-changing the appropriate subscripts) are given by

$$\Pi_1^{JM-HB} = (p_A - c)s_A(p) + (p_B - c - t)s_B(p) \tag{9}$$

After substituting in the prices that would yield zero surplus for the marginal consumer, and expressing market shares in terms of the location of the marginal consumer, we obtain

$$\Pi_1^{JM-HB} = (V + h - \theta x_c - c)x_c + (V + \theta x_c - \theta - x - t)(1 - x_c) \tag{10}$$

Maximizing with respect to x_c we obtain the first order condition

$$h - 3\theta x_c + \theta(1 - x_c) + \theta + t = 0 \tag{11}$$

which solving the x_c yields

$$\tilde{x}_1^{JM-HB} = \frac{1}{2} + \frac{1}{4}\frac{t+h}{\theta} \tag{12}$$

Notice that an increase in h increases $\tilde{x}_1^{JM-HB}$ by more than it increases $\tilde{x}_1^{C-HB}$. Given that $\tilde{x}_1^{JM-HB}$ is to the right of $\tilde{x}_1^{C-HB}$ for $h = 0$, home bias increases the cross-hauling in the non-cooperative equilibrium compared to the cross-hauling under collusion.

Finally, let us compute the socially optimal cross-hauling as a function fo h. Following the insight on the social planner's problem, the socially optimal price difference in the price of the imported and domestic variants will be equal to t regardless of the value of h. Therefore, the location of the indifferent consumer in the first best is given by[35]

$$\tilde{x}_1^{FB-HB} = \frac{1}{2} + \frac{1}{2}\frac{t+h}{\theta} \tag{13}$$

Notice that an increase in h increases $\tilde{x}_1^{FB-HB}$ by more than either $\tilde{x}_1^{JM-HB}$ or $\tilde{x}_1^{C-HB}$. Given that $\tilde{x}_1^{FB-HB}$ is the right-most of the three critical values, and increase in h increases cross-hauling under both the competitive and the collusive equilibria relative to the first-best. The intuition as to why the socially optimal level of cross-hauling is more sensitive to the non-cooperative level of cross-hauling is clear. The welfare maximizing price difference between the domestic and imported good is independent of h and equal to t. The non-cooperative price difference between the two prices is increasing in h (see above). The increase in the price gap partially mitigates the increase in the willingness to pay for the domestic good, and thus dampens the effect that h has on market shares. The effect of h on the collusive market division is intermediate to the socially optimal and competitive regimes, as was the case for $h = 0$.

It is worth noting that the home bias operates through a different channel than the physical transport costs across the two markets. The latter increase all prices in the competitive equilibrium (albeit not symmetrically), while a planner would increase only the price of the imported good. The former increases the domestic price and decreases the imported price, while a planner would not change prices at all. Thus, both home bias and cross-hauling costs create distortions by introducing asymmetries, but one operates from the supply side while the other operates from the demand side. Of course, cross-hauling is empirically more salient as it applies to essentially all goods traded across markets, while home bias much less so, as it applies to a sub-set of consumer goods that are traded across international borders.

[35] The same result can be obtained by starting from first principles and maximizing the social welfare function.

4 Concluding remarks

As Kaplow and Shapiro (2007) point out, "(c)olluding firms use a variety of methods to achieve the basic goal of raising prices. In some cases, firms agree to minimum prices. In others, they agree to limit their production levels, since output restrictions translate into elevated prices. Alternatively, firms can allocate customers or territories among themselves, with each firm agreeing not to compete for customers, or in territories, assigned to others." (p.1099). Our paper has been concerned with the simultaneous effects of multimarket coordination on prices, quantity traded and welfare in a differentiated-goods industry.

In the context of spatial industries where aggregate demand effects are small, we have provided a model where the following clear-cut results obtain: (i) (perfect) collusion reduces, though does not eliminate, trade relative to competition, leading to a cartel allocation consistent with the "home-market principle"; (ii) this collusive reduction in trade (and thus reduction in the heterogeneity of consumption) enhances total welfare; (iii) the welfare gain from collusion occurs even when the trade cost is low (with this welfare gain increasing in the trade cost and decreasing in the degree of product differentiation); (iv) the cartel's reduction in trade can even enhance *consumer* welfare relative to the competitive regime; (v) even collusion involves some degree of excessive trade relative to the welfare optimum; and (vi) for a sufficiently high trade cost, even the (extreme) prohibition of trade—i.e. imposing autarky—improves welfare over the market-based competitive and collusive regimes.

These results have direct implications for anti-trust policy on the co-ordination of geographically separated suppliers. Their importance for cross-border co-ordinations is of particular note; we believe they might be useful in assessing public policy with regards to international cartels and (more generally) with regards to trade in differentiated products. Beyond the confines of this particular modeling framework, we believe that our results are but one example of in which collusion can result in Pareto superior outcomes both for the firms involved and for the consumers even in the absence of any direct transfers between the firms. What is particularly interesting in our framework is that we obtain this win-win possibility result even in the absence of any asymmetry of the firms across all markets. The welfare reducing distortion that the cartel eliminates arises from the comparative advantage of the firms to supply particular markets. We believe that a more systematic examination of conditions that collusion can yield such Pareto superior outcomes in the absence of transfers and firm differences (in the aggregate) is an interesting research agenda.

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A Appendix: Sustaining the fully collusive outcome

As mentioned, firms are assumed to sustain the fully collusive outcome by adopting grim trigger strategies that account for the multimarket nature of their contact. Total firm profits in the collusive regime and the competitive (punishment) regime are given respectively by $\Pi_{1A}^{JM} + \Pi_{2A}^{JM}$ and $\Pi_{1A}^{C} + \Pi_{2A}^{C}$. Recall that for the entire space of parameters restricted by A1 and A2, it is always the case that both firms command positive market shares in each of the two markets both along the equilibrium path (full collusion) and during punishment periods (competitive Nash equilibrium). The same is not always true, however, with regard to a defection period.

If a firm were to cheat on the collusive agreement, it would clearly be optimal to cheat simultaneously in both local markets, since punishment does not depend on the degree of defection. Now, in the parameter space we consider, there are defection-period cases where a deviant firm captures the entire home market but less than full share of the foreign market, or where the deviant firm ends up capturing both markets entirely. In what follows, we first analyze optimal deviation (conditional on a firm deviating from the equilibrium path). Since marginal cost is flat in output, defection can be analyzed separately for the home market and the away market. Subsequently, we study the perfect cartel's incentive constraint in each deviation case. Without loss of generality, our deviant firm is firm A.

A.1 Optimal deviation in the home market

In market 1, if firm B abides by the agreement and sets a price of p_{1B}^{JM} and firm A deviates and sets a price of p_{1A}^{D}, deviant firm A's quantity share is $s_{1A}(p_{1A}^{D}, p_{1B}^{JM})$. There are two possible scenarios for the defection period. First, both firms might have positive market shares, i.e. $s_{1A}(p_{1A}^{D}, p_{1B}^{JM}) < 1$. Second, it may be that firm A's deviation completely prices firm B out of the market.

First scenario: Both firms have positive market shares in the defection period. Solving deviant firm A's home-market optimization problem

$$\max_{p} (p - c)\, s_{1A}(p, p_{1B}^{JM})$$

yields its optimal deviation price, profit and quantity share, respectively:

$$\begin{aligned} p_{1A}^{D} &= \frac{1}{2}V + \frac{1}{4}\theta + \frac{1}{2}c + \frac{1}{8}t \\ \Pi_{1A}^{D} &= \frac{1}{128\theta}\left(4V + 2\theta - 4c + t\right)^2 \\ s_{1A}^{D} &= \frac{1}{16\theta}\left(4V + 2\theta - 4c + t\right) \end{aligned} \tag{14}$$

for combinations of parameters that satisfy $s_{1A}^{D} < 1 \Leftrightarrow 4V - 14\theta - 4c + t < 0$.

Second scenario: Firm A captures the entire market in the defection period. If instead $4V - 14\theta - 4c + t \geq 0$, then deviant firm A sets price p_{1A}^{D} such that the consumer located at $x_1 = 1$ is indifferent between goods A and B, i.e. $s_{1A}(p_{1A}^{D}, p_{1B}^{JM}) = 1$. This implies that

$$p_{1A}^{D} = V - \frac{3}{2}\theta + \frac{1}{4}t$$

and thus optimal deviation profit is:

$$\Pi_{1A}^{D} = V - \frac{3}{2}\theta - c + \frac{1}{4}t \tag{15}$$

A.2 Optimal deviation in the away market

Similarly, in market 2, deviant firm A's quantity share when setting price p_{2A}^{D} (and firm B abides by the cartel agreement) is $s_{2A}(p_{2A}^{D}, p_{2B}^{JM})$. Again, there are two scenarios for the defection period.

First scenario: Both firms have positive market share in the defection period. Solving deviant firm A's away-market optimization problem

$$\max_{p}\,(p - c - t)\, s_{2A}(p, p_{2B}^{JM})$$

yields respective deviation price, profit and quantity share:

$$p_{2A}^{D} = \frac{1}{2}V + \frac{1}{4}\theta + \frac{1}{2}c + \frac{3}{8}t$$

$$\Pi_{2A}^{D} = \frac{1}{128\theta}\,(4V + 2\theta - 4c - 5t)^2 \tag{16}$$

$$s_{2A}^{D} = \frac{1}{16\theta}\,(4V + 2\theta - 4c - 5t)$$

for combinations of parameters that satisfy $s_{2A}^{D} < 1 \Leftrightarrow 4V - 14\theta - 4c - 5t < 0$. This condition is clearly less stringent than $s_{1A}^{D} < 1$ above: intuitively, it may be that the deviant firm does not capture the entire foreign market (i.e. $4V - 14\theta - 4c - 5t < 0$ is satisfied) but does capture the entire home market ($4V - 14\theta - 4c + t < 0$ is not satisfied).

Second scenario: Firm A captures the entire market in the defection period. If instead $4V - 14\theta - 4c - 5t \geq 0$, then deviant firm A sets price p_{2A}^{D} such that the consumer located at $x_2 = 1$ is indifferent between goods A and B, i.e. $s_{2A}(p_{2A}^{D}, p_{2B}^{JM}) = 1$. (Analogously, $4V - 14\theta - 4c - 5t \geq 0$ is more stringent than $4V - 14\theta - 4c + t \geq 0$, the above condition for the home market's second defection scenario: intuitively, if the deviant firm captures the entire foreign market, this implies that it captures the entire home market.) Optimal deviation price and profit are

$$p_{2A}^{D} = V - \frac{3}{2}\theta - \frac{1}{4}t$$

$$\Pi_{2A}^{D} = V - \frac{3}{2}\theta - c - \frac{5}{4}t \qquad (17)$$

A.3 Supporting the fully collusive outcome

If both firms play 'collude' then each firm earns a total profit of Π^{JM} (across the two markets, per period); if firms 'compete' then each firm earns a total profit of Π^{C}; and if a firm 'deviates' when the other firm plays 'collude' then the deviator earns a total profit of Π^{D}. As mentioned above, in sustaining collusion, the grim trigger strategy is used so that any deviation results in the competitive solution (that is, static Nash equilibrium) forever. The incentive compatibility constraint (ICC) for collusion is then:

$$\Pi^{JM}\frac{1}{1-\delta} \geq \Pi^{D} + \Pi^{C}\frac{\delta}{1-\delta}$$

or, equivalently,

$$\delta \geq \frac{\Pi^{D} - \Pi^{JM}}{\Pi^{D} - \Pi^{C}} \equiv \delta^{*} \qquad (18)$$

where $\delta \in (0,1)$ denotes the firms' common discount factor, and Π^{C} and Π^{JM} follow from (3) and (5):

$$\Pi^{JM} = \Pi_{1A}^{JM} + \Pi_{2A}^{JM} = \frac{4\theta\left(2V - \theta - 2c - t\right) + t^{2}}{8\theta} \qquad (19)$$

and

$$\Pi^{C} = \Pi_{1A}^{C} + \Pi_{2A}^{C} = \frac{9\theta^{2} + t^{2}}{9\theta} \qquad (20)$$

With regard to the period of defection, three different cases should be considered which we discuss in turn.

Case I: Both firms command positive market shares in each of the two markets: $4V - 14\theta - 4c + t < 0$ The deviant firm's defection-period profit across both markets is then the sum of (14) and (16):

$$\Pi^D\big|_{\text{Case I}} = \frac{(4V + 2\theta - 4c + t)^2 + (4V + 2\theta - 4c - 5t)^2}{128\theta} \tag{21}$$

So, making use of eqs. (18)-(21), the critical discount factor above which the fully collusive outcome can be sustained in this case is given by:

$$\delta^*\big|_{\text{Case I}} = 9\frac{16\,(V-c)\,(V-c-t) - 48\theta\,(V-c) + 12\theta\,(3\theta+2t) + 5t^2}{144\,(V-c)\,(V+\theta-c-t) - 36\theta\,(15\theta+2t) + 53t^2}$$

Case II: The deviant firm captures the entire home market but less than full share of the foreign market: $4V - 14\theta - 4c + t \geq 0$ **and** $4V - 14\theta - 4c - 5t < 0$**.** The defection-period profit across both markets is the sum of (15) and (16):

$$\Pi^D\big|_{\text{Case II}} = \left(V - \frac{3}{2}\theta - c + \frac{1}{4}t\right) + \frac{1}{128\theta}\,(4V + 2\theta - 4c - 5t)^2 \tag{22}$$

Therefore, making use of eqs. (18)-(20) and (22), the critical discount factor for this case is:

$$\delta^*\big|_{\text{Case II}} = 9\frac{16\,(V-c)\,(V+\theta-c) - 40t\,(V-c) + 4\theta\,(19t-31\theta) + 9t^2}{72\,(V-c)\,(2V+18\theta-2c-5t) + (97t-474\theta)\,(6\theta+t)}$$

Case III: The deviant firm captures not only the home market but also the foreign market entirely: $4V - 14\theta - 4c - 5t \geq 0$ The defection-period profit across both markets is the sum of (15) and (17):

$$\Pi^D\big|_{\text{Case III}} = 2V - 3\theta - 2c - t \tag{23}$$

So, making use of eqs. (18)-(20) and (23), the critical discount factor for this case is:

$$\delta^*\big|_{\text{Case III}} = \frac{9}{8}\,\frac{8\theta\,(V-c) - 4\theta\,(5\theta+t) - t^2}{18\theta\,(V-c) - 9\theta\,(4\theta+t) - t^2}.$$

B Appendix: Proof of Proposition 5 (and related statements)

We start by considering the collusive regime. With the tax and subsidy, the perfect cartel's univariate problem of Section 2.2 changes to

$$\max_{p_A} (p_A - c + \omega) \frac{V - p_A}{\theta} + (2V - \theta - p_A - c - t - \tau) \left(1 - \frac{V - p_A}{\theta}\right)$$

(For brevity, we write the proof for an interior solution, $t < \theta$; otherwise the equilibrium outcome is the corner solution of Section 3.2.) This yields prices and profits

$$p_{1A}^{JM}(\tau, \omega) = V - \frac{1}{2}\theta - \frac{1}{4}(t + \tau + \omega), \quad p_{1B}^{JM}(\tau, \omega) = 2V - \theta - p_{1A}^{JM}(\tau, \omega) = V - \frac{1}{2}\theta + \frac{1}{4}(t + \tau + \omega)$$

$$\Pi_{1A}^{JM}(\tau, \omega) = \frac{1}{16\theta}(2\theta + t + \tau + \omega)(4V - 2\theta - 4c - t - \tau + 3\omega)$$

$$\Pi_{1B}^{JM}(\tau, \omega) = \frac{1}{16\theta}(2\theta - t - \tau - \omega)(4V - 2\theta - 4c - 3t - 3\tau + \omega)$$

The marginal consumer, who has zero surplus, is now located at:

$$\tilde{x}_1^{JM}(\tau, \omega) = \frac{1}{2} + \frac{1}{4}\frac{t + \tau + \omega}{\theta}$$

For the first-best market allocation to attain, i.e. $\tilde{x}_1^{JM}(\tau, \omega) = \tilde{x}_1^{FB}$, it is clear from Section 3.2 that $(t + \tau + \omega)/(4\theta) = t/(2\theta)$ and thus a necessary condition is

$$\tilde{x}_1^{JM} \to \tilde{x}_1^{FB}: \qquad \tau + \omega = t$$

(Alternatively, one can compute, as in Section 3.1, the sum of the total cost of cross-hauling and the total consumer taste disutility, $t\left(1 - \tilde{x}_1^{JM}(\tau, \omega)\right) + \int_0^{\tilde{x}_1^{JM}(\tau,\omega)} \theta x dx + \int_{\tilde{x}_1^{JM}(\tau,\omega)}^1 \theta (1 - x) dx$, and minimize this sum with respect to the policy instruments.) As expected, $p_{1B}^{JM}(\tau, \omega) - p_{1A}^{JM}(\tau, \omega) =^{\tau+\omega=t} t$.

To verify the example provided in the text, $(\tau, \omega) = (t, 0)$, notice that by construction of the cartel's univariate problem, the marginal consumer's utility is zero, while profits on both the home good and the imported good are non-negative. (To see this, notice that $\Pi_{1A}^{JM}(\tau = t, \omega = 0) = (\theta + t)(2V - \theta - 2c - t)/(4\theta) >^{\text{A2}} 0$ and $\Pi_{1B}^{JM}(\tau = t, \omega = 0) = (\theta - t)(2V - \theta - 2c - 3t)/(4\theta) >^{t<\theta} (\theta - t)(2V - 3\theta - 2c - t)/(4\theta) \geq^{\text{A2}} 0$.) So individual rationality constraints are satisfied.

Now consider the competitive regime. In the presence of a tax and a subsidy, prices in the competitive equilibrium solve the modified system (*cf.* Section 2.1)

$$\begin{cases} \max\limits_{p_A} (p_A - c + \omega)\, s_A(p) \\ \max\limits_{p_B} (p_B - c - t - \tau)\, s_B(p) \end{cases}$$

yielding prices and profits (again, for brevity, we write the proof for an interior solution, $t < \theta$, otherwise the corner solution of Section 3.2 applies)

$$p_{1A}^C(\tau,\omega) = \theta + c + \frac{1}{3}(t + \tau - 2\omega)\,, \quad p_{1B}^C(\tau,\omega) = \theta + c + \frac{1}{3}(2t + 2\tau - \omega)$$

$$\Pi_{1A}^C(\tau,\omega) = \frac{1}{18\theta}(3\theta + t + \tau + \omega)^2\,, \quad \Pi_{1B}^C(\tau,\omega) = \frac{1}{18\theta}(3\theta - t - \tau - \omega)^2$$

The equilibrium location of the marginal consumer is given by

$$\tilde{x}_1^C(\tau,\omega) = s_{1A}(p(\tau,\omega)) = \frac{1}{2} + \frac{1}{6}\frac{t + \tau + \omega}{\theta}$$

Similar to the above, for the first-best market allocation to attain, i.e. $\tilde{x}_1^C(\tau,\omega) = \tilde{x}_1^{FB}$, it follows that $(t + \tau + \omega)/(6\theta) = t/(2\theta)$ and thus a necessary condition is

$$\tilde{x}_1^C \to \tilde{x}_1^{FB}: \qquad \tau + \omega = 2t$$

Note, similarly, that $p_{1B}^C(\tau,\omega) - p_{1A}^C(\tau,\omega) =^{\tau+\omega=2t} t$.

Now verify that individual rationality constraints are satisfied in the example provided in the text, $(\tau,\omega) = (2t/3, 4t/3)$. The marginal consumer's utility is $U_A(p_{1A}^C(2t/3, 4t/3); \tilde{x}_1^{FB}) = \frac{1}{2}\left(2V - 3\theta - 2c - \frac{1}{3}t\right) > \frac{1}{2}(2V - 3\theta - 2c - t) \geq^{\text{A2}} 0$.[36] Profits on both the home good and the imported good are clearly positive.

Still considering the competitive regime, for the cartel market allocation to attain, i.e. $\tilde{x}_1^C(\tau,\omega) =$

[36] In the competitive regime, the tax-only policy along the $\tau + \omega = 2t$ locus, i.e. $(\tau,\omega) = (2t, 0)$, would not necessarily be consistent with full market coverage. To see this, notice that $U_A(p_{1A}^C(2t,0); \tilde{x}_1^{FB}) = \frac{1}{2}(2V - 3\theta - 2c - 3t) < \frac{1}{2}(2V - 3\theta - 2c - t)$; so if A2 is within $2t$ of binding, such a policy would result in incomplete market coverage.

$\tilde{x}_1^{JM}$, a necessary condition is $(t+\tau+\omega)/(6\theta) = t/(4\theta)$ or

$$\tilde{x}_1^C \to \tilde{x}_1^{JM}: \qquad \tau+\omega = \frac{1}{2}t$$

where, as expected, $p_{1B}^C(\tau,\omega) - p_{1A}^C(\tau,\omega) =^{\tau+\omega=t/2} p_{1B}^{JM} - p_{1A}^{JM} = \frac{1}{2}t$.

Verifying the example provided, $(\tau,\omega) = (\frac{1}{4}t, \frac{1}{4}t)$, profits on both home and imported goods are similarly positive, and the marginal consumer's utility is $U_A(p_{1A}^C(\frac{1}{4}t, \frac{1}{4}t); \tilde{x}_1^{JM}) = \frac{1}{2}(2V - 3\theta - 2c - t) \geq^{\text{A2}} 0$. Thus individual rationality constraints are met. For this example, both consumer surplus and firm profits turn out to be higher relative to the tax-and-subsidy-free competitive outcome. Calculating $CS^C(\frac{1}{4}t, \frac{1}{4}t) = \int_0^{\tilde{x}_1^{JM}} \left(V - \theta x - p_{1A}^C(\frac{1}{4}t, \frac{1}{4}t)\right) dx + \int_{\tilde{x}_1^{JM}}^1 \left(V - \theta(1-x) - p_{1B}^C(\frac{1}{4}t, \frac{1}{4}t)\right) dx$ and subtracting CS^C (as mentioned in Section 3.1, CS is the area of two trapezia), it follows that $CS^C(\frac{1}{4}t, \frac{1}{4}t) - CS^C = \frac{5}{144}\frac{t^2}{\theta} > 0$. Also, each firm's profit increases by $\Pi_{1A}^C(\frac{1}{4}t, \frac{1}{4}t) + \Pi_{1B}^C(\frac{1}{4}t, \frac{1}{4}t) - \Pi_{1A}^C - \Pi_{1B}^C = \frac{5}{36}\frac{t^2}{\theta} > 0$. ■

C Appendix: Proof of Proposition 6

We begin by further examining, within the space of parameters defined by A1 and A2, the subspace where full market coverage obtains in autarky (as shown in the text, this occurs if $V \geq 2\theta + c$) from the subspace where coverage in autarky is incomplete (i.e. $V < 2\theta + c$). Consider A1: $(0 <)t < 2\theta$. Notice that a sufficient condition for full coverage in autarky is $t \geq \theta$, since $V \geq^{\text{A2}} \frac{1}{2}(3\theta + 2c + t) \geq^{t \geq \theta} 2\theta + c$. For $t < \theta$, full coverage in autarky may or may not obtain, as $\frac{1}{2}(3\theta + 2c + t) <^{t<\theta} 2\theta + c$ and thus $V \gtreqless 2\theta + c$ (e.g. say that t is low and A2 almost binds: then $2V \simeq 3\theta + 2c < 4\theta + 2c$, so in autarky there is incomplete coverage). Notice that incomplete coverage ($V < 2\theta + c$) implies that $t \leq^{\text{A2}} 2V - 3\theta - 2c <^{V<2\theta+c} \theta$.

Now compute (per-market) social welfare under autarky. Consider the first case, of full coverage: $V \geq 2\theta + c$. Consumer surplus is $\frac{1}{2}\theta$ (the area of a triangle with height $V - p^{AUTK} = \theta$ and unit width) and producer surplus is $\Pi^{AUTK} = V - \theta - c$, the sum of which yields social welfare: $W^{AUTK} = V - \frac{1}{2}\theta - c$. Next consider the complementary case, of incomplete coverage: $V < 2\theta + c$. Consumer surplus is $(V-c)^2/(8\theta)$ (the area of a triangle with height $V - p^{AUTK} = \frac{1}{2}(V-c)$ and width $\check{x}^{AUTK} = (V-c)/(2\theta)$) and producer surplus is $\Pi^{AUTK} = (V-c)^2/(4\theta)$, with welfare totaling $W^{AUTK} = 3(V-c)^2/(8\theta)$.

Next compute social welfare under each of the two market-based trade regimes. (Recall that in these regimes, throughout the space of parameters, the market is fully covered and cross-hauling occurs.) Consumer surplus is calculated as explained in Section 3.1 (equivalent to the area of two trapezia, which in the collusive regime collapses to the area of two triangles as the marginal consumer has zero surplus). Producer surplus is given by the sum of the home firm's profit on home sales and its profit on foreign sales, as stated in Sections 2.1 (competitive regime) and 2.2 (collusive regime). For brevity, we simply state the sum of consumer surplus and producer surplus in each regime: $W^C = \left(36V\theta - 36c\theta - 18t\theta - 9\theta^2 + 5t^2\right) / (36\theta)$ and $W^{JM} = \left(16V\theta - 16c\theta - 8t\theta - 4\theta^2 + 3t^2\right) / (16\theta)$.

We now calculate welfare differences across regimes, first considering the parameter subspace for which there is full coverage under autarky (i.e. $V \geq 2\theta + c$). We compute $16\theta\left(W^{AUTK} - W^{JM}\right) = -3t^2 + 8t\theta - 4\theta^2$ which, being concave in t and having roots $t = \frac{2}{3}\theta, 2\theta$, is strictly positive over the interval $\frac{2}{3}\theta < t < 2\theta$: hence (conditional on full coverage, and recalling Proposition 2) we have $W^{AUTK} > W^{JM} > W^C$. This proves statements (i) and (ii). Further, proof of the second part of statement (iv) follows from noting that $-3t^2 + 8t\theta - 4\theta^2$ is negative for $t < \frac{2}{3}\theta$, but increasing in t. Similarly, we compute $36\theta\left(W^{AUTK} - W^C\right) = -5t^2 + 18t\theta - 9\theta^2$ which is strictly positive over the interval $\frac{3}{5}\theta < t < 3\theta$. So for $\frac{3}{5}\theta < t \leq \frac{2}{3}\theta$ (and full coverage) we have $W^{JM} \geq W^{AUTK} > W^C$, proving statement (iii). Proof of the first part of statement (iv) follows, similarly, from noting that, for $t \leq \frac{3}{5}\theta$, $-5t^2 + 18t\theta - 9\theta^2$ is (weakly) negative and increasing in t.

It remains to prove statement (v), pertaining to the parameter subspace for which market coverage in autarky is incomplete (i.e. $V < 2\theta + c$, implying that $t \leq 2V - 3\theta - 2c < \theta$). We compute $16\theta\left(W^{AUTK} - W^{JM}\right) = 2(V - c)(3V - 8\theta - 3c) + \left(-3t^2 + 8t\theta + 4\theta^2\right)$. Since $V - c >^{\text{A2}} 0$ and, conditional on incomplete coverage, $V - 2\theta - c < 0 \Leftrightarrow 3V - 6\theta - 3c < 0 \Rightarrow 3V - 8\theta - 3c < 0$, the first bunch of terms is negative. It is also invariant in t. Noting that, over the interval $0 < t \leq 2V - 3\theta - 2c < \theta$, the parabola defined by $-3t^2 + 8t\theta + 4\theta^2$ is positive and increasing in t, it follows that $W^{AUTK} - W^{JM} \lesseqgtr 0$ and that $W^{AUTK} - W^{JM}$ increases in t. Similarly, we compute $16\theta\left(W^{AUTK} - W^C\right) = 2(V - c)(3V - 8\theta - 3c) + \frac{4}{9}\left(-5t^2 + 18t\theta + 9\theta^2\right)$ where the (same t-invariant) first bunch of terms is negative and, over the interval $0 < t \leq 2V - 3\theta - 2c < \theta$, the parabola $\frac{4}{9}\left(-5t^2 + 18t\theta + 9\theta^2\right)$ is positive (in fact, consistent with Proposition 2, larger than $-3t^2 + 8t\theta + 4\theta^2$) and increasing in t. It follows that $W^{AUTK} - W^C \lesseqgtr 0$ and that $W^{AUTK} - W^C$ increases in t. This proves (v). In this subspace of incomplete coverage under autarky, we further show that as $t \to 0^+$, $\left(W^{AUTK} - W^{JM} <\right) W^{AUTK} - W^C < 0$. The left inequality follows from Proposition 2. The right inequality follows from noting that as $t \to 0^+$, $8\theta\left(W^{AUTK} - W^C\right) \to (V - c)(3V - 8\theta - 3c) + 2\theta^2 < 0$. To see this, notice that $(V - c)(3V - 8\theta - 3c) + 2\theta^2 < 0 \Leftrightarrow -(3V - 8\theta - 3c)(V - c) > 2\theta^2$, and that $V < 2\theta + c \Leftrightarrow -(3V - 8\theta - 3c) > 2\theta > 0$ and $V - c \geq^{\text{A2}}$

$\frac{3}{2}\theta + \frac{1}{2}t > \theta > 0$. Also in this subspace of incomplete coverage under autarky, we show (by example) that as $t \to 2V - 3\theta - 2c < \theta$, $W^{AUTK} - W^{C}$ remains negative (e.g. $V = 3, \theta = 1, c = 1.2$) or can become positive (e.g. $V = 3, \theta = 1, c = 1.05$). Similarly, as $t \to 2V - 3\theta - 2c < \theta$, $W^{AUTK} - W^{JM} \left(< W^{AUTK} - W^{C}\right)$ remains negative or can become positive (see the same respective examples). ■

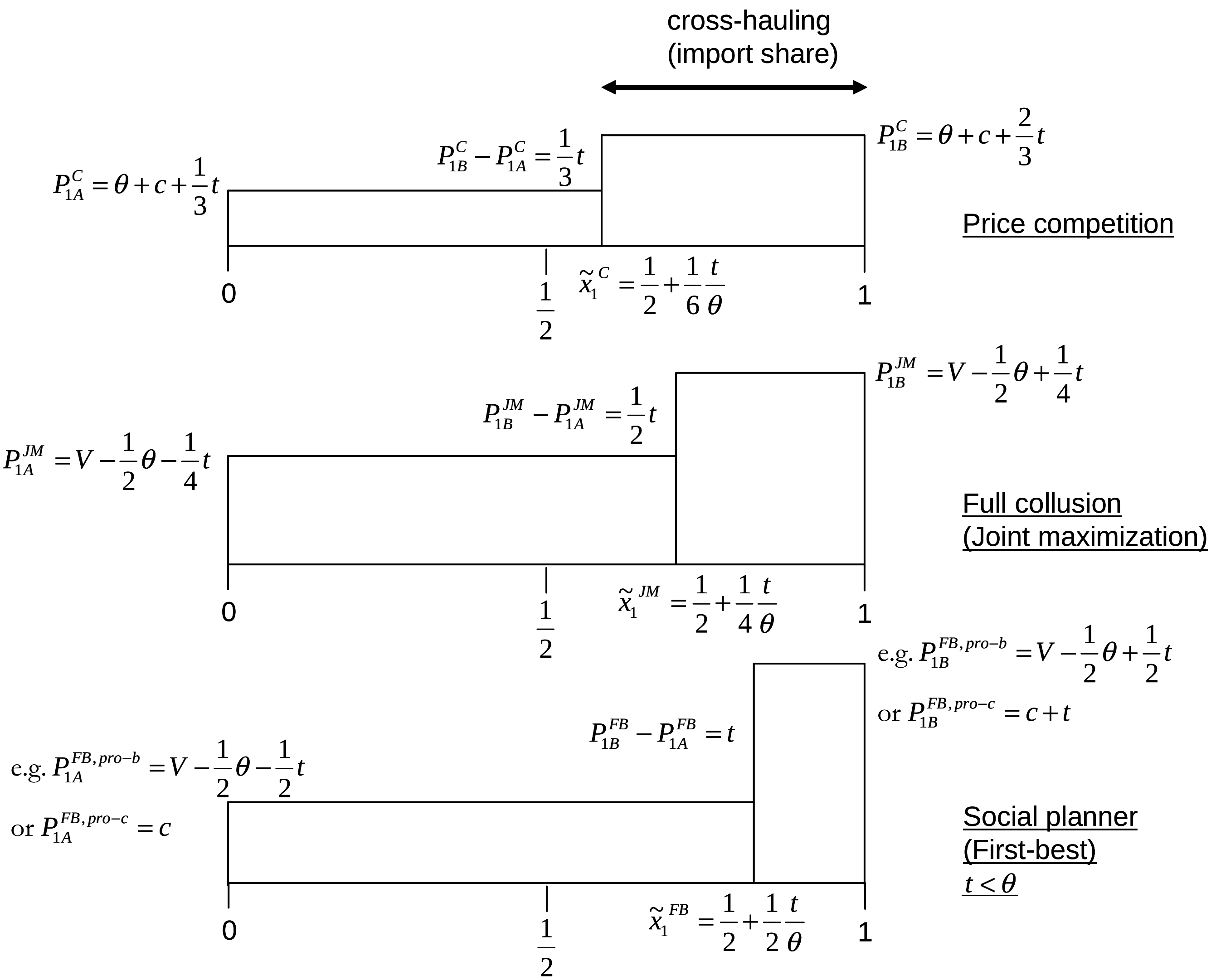

Figure 1: Quantity shares and prices for the home good and the foreign good in market 1, for different trade regimes (in the restricted space of parameters): Price competition (top panel), full collusion (middle panel), and the socially first-best outcome (bottom panel). Market 2 is symmetric. The bottom panel is drawn for $t < \theta$.

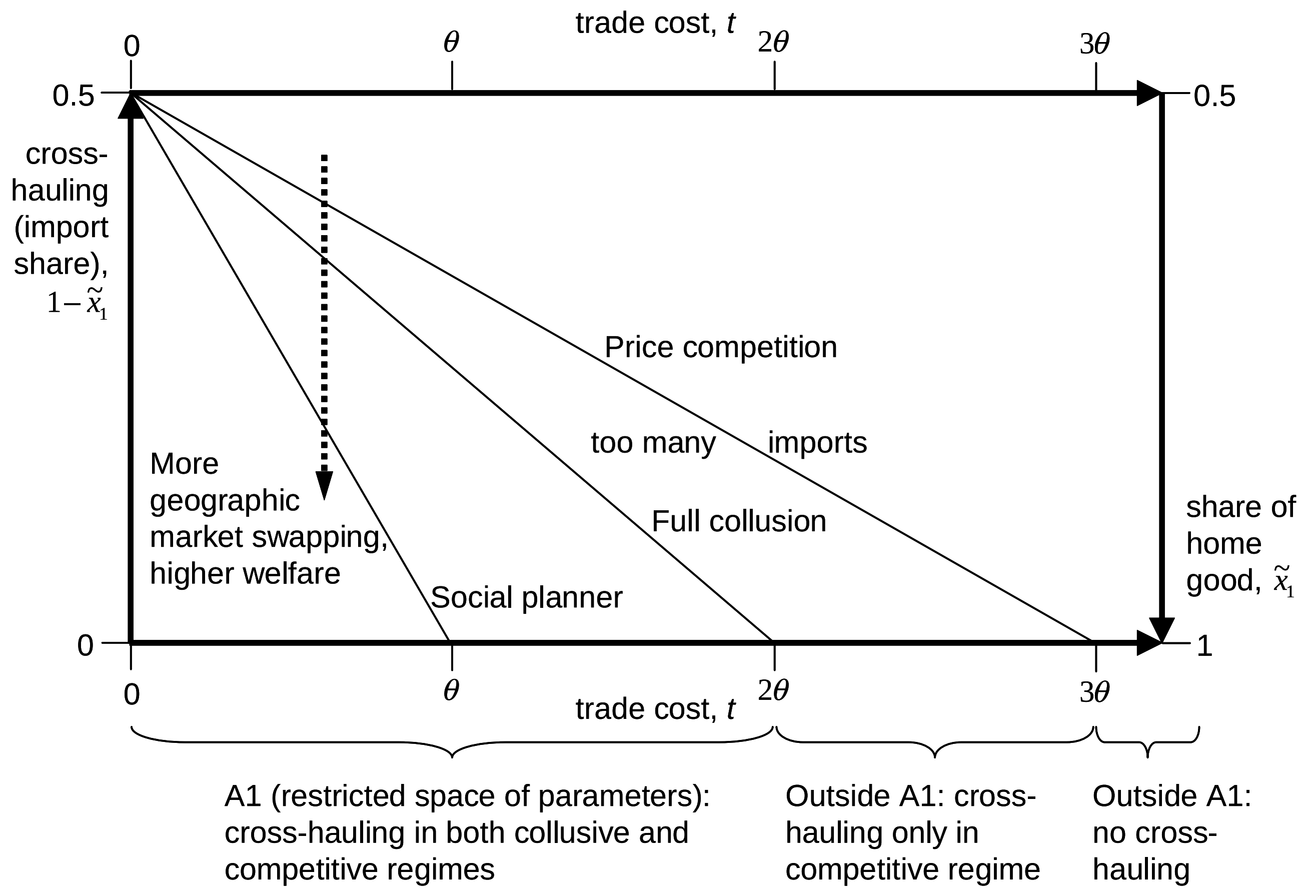

Figure 2: The extent of cross-hauling across the different trade regimes, within *and* beyond the restricted space of parameters (A1). Market 1's import share $1 - \tilde{x}_1$ (left axis) and share of home good $\tilde{x}_1$ (right axis, inverted scale) against trade cost t.