Market Segmentation Strategies of Multiproduct Firms*

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Abstract

We analyze a multiproduct duopoly and ask whether firms should offer general purpose products or tailor their offerings to fit specific consumer needs. Offering a targeted product has two effects: utility increases for some consumers due to increased *fit*, whereas utility decreases for others due to increased *misfit*. Previous work has not considered these two effects jointly and has therefore not been able to capture the tradeoff inherent in market segmentation. We show that in addition to the degree of fit and misfit, the intensity of competition and the fixed cost of offering an additional product determine firms' market segmentation strategies.

1 Introduction

Extending product lines to target more narrowly defined consumer segments has been a favorite strategy of brand managers for years. Popular buzzwords like *one-on-one marketing* and *niche marketing* underline its importance in practice. In this context,

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we ask to what extent market segmentation is justified from a profitability point of view.

Consider a competitive environment in which each firm can either offer a general purpose product or segment the market by offering products that are tailored to consumers' needs. While a consumer *ceteris paribus* prefers a product that is targeted at her own segment, she would rather buy a general purpose product than a product that is targeted at another segment. Segmenting the market therefore has two effects. First, if a consumer's preferred product is offered, she is better off because she is able to buy a product that exactly fits her needs. We call this positive aspect of market segmentation *fit*. Second, if the consumer's preferred product is not offered, she is worse off because she ends up with a product that does not satisfy her needs at all. This negative aspect of market segmentation is denoted as *misfit*.

For example, in the sport shoe industry, a firm can either offer a general purpose cross-trainer shoe or a running shoe and a basketball shoe. A cross-trainer shoe has wide appeal for all consumers but satisfies no consumer's needs in particular, while the specific shoes each satisfy the needs of a particular segment of consumers but have little appeal for the other segment. On the other hand, the firm can offer both a running shoe and a basketball shoe. Manufacturing a larger number of products however carries a greater fixed cost than manufacturing a smaller number of products. Whether the firm will follow a niche or a full-line strategy therefore depends on the cost of offering an additional product and the revenue generated by doing so. The revenue in turn depends on the intensity of competition in the market.

In this paper we investigate how consumers' preferences, firms' cost structures, and the strategic interaction of firms in the product market together shape firms' market segmentation decisions. There are two streams of literature that are related to our research: the literature on spatial competition and the one on multiproduct firms. We discuss each of them in turn.

In models of spatial competition (horizontal product differentiation) misfit or *specificity* is operationalized by transportation costs and fit or *quality* by gross benefits. The existing research has studied these two aspects of market segmentation separately from each other. For example, von Ungern-Sternberg (1988) considers n single-product firms and analyzes a two-stage game in which entry decisions are followed by choice of transportation costs and prices. He finds that the private incentives to produce general purpose products (i.e., lower transportation costs) are excessive relative to the social optimum. Hendel & Neiva de Figueiredo (1998) argue that von Ungern-Sternberg's (1988) timing prevents the choice of transportation costs to have a strategic effect on firms' pricing and hence propose a three-stage game: entry decisions, followed by choice of transportation costs, and then choice of prices. Their results suggest that when the degree of specificity can be varied without affecting firms' costs, then at most two firms will enter the market and choose positive transportation costs. The results for the case where general purposeness is costly are inconclusive.[1]

Rather than focusing on endogenous transportation costs, Economides (1989, 1993) analyzes endogenous gross benefits and interprets them as quality. Economides (1993) finds too much variety and too little quality in a free-entry equilibrium. Unlike the above models, which all consider competition on a circle, Economides (1989) has consumers located along a line. He explicitly considers firms' location decisions and shows that the ensuing equilibrium has maximum location differentiation but minimum quality differentiation.

None of these models captures the tradeoff between the increased fit for some

[1]Dos Santos Ferreira & Thisse (1996) study a two-stage game in which prices are chosen after transportation costs. If firms are located near the center (end points) of a Hotelling line, then they choose maximal (minimal) transportation costs.

consumers and the increased misfit for others that is inherent in market segmentation. In particular, by equating general purposeness with transportation costs, these models miss out on the idea that a targeted product makes some – but not all – consumers better off. Similarly, a change in gross benefits is assumed to affect all consumers in the same way. More important, these models are ill-suited to study firms' market segmentation decisions because each firm only offers a single product.

Brander & Eaton (1984) first analyzed competition between multiproduct duopolists. Constraining each firm to two products and fixing the attributes of the four products in the market, they ask whether a firm would decide to produce a pair of close substitutes (market segmentation) or a pair of distant substitutes (market interlacing). Their assumption that a firm has to offer two products has subsequently been criticized. In particular, Martinez-Giralt & Neven (1988) argue that when product attributes are endogenized, price competition gives rise to Hotelling's principle of maximum differentiation and *de facto* prevents multiproduct firms from arising. In the context of the sport shoe industry, this suggests an outcome of market segmentation with niche firms, i.e., one firm offers a running shoe, the other a basketball shoe. What we observe in practice however is that a number of firms compete head-on by offering multiple products.

In contrast to models of spatial competition, multiproduct firms can be the outcome of models of quality competition (vertical product differentiation). Champsaur & Rochet (1989) show that firms' desire to avoid head-on competition leads to a gap between the product offerings of the duopolists, i.e., one firm offers a range of low-quality products, the other a range of high-quality products. Gilbert & Matutes (1993) allow consumers to have idiosyncratic preferences for firms and show that firms may adopt a full-line rather than a niche strategy. The driving force behind multiproduct firms is that in models of quality competition, a firm can better engage in second-degree price discrimination by offering a larger number of products. In a setting with horizontal product differentiation like ours, by contrast, price discrimination is not an issue. In fact, we rule out price discrimination altogether by assuming that a firm charges the same price for all its products. Multiproduct firms thus cannot arise for the same reason as in models of quality competition.

To investigate firms' market segmentation strategies in the context of horizontal product differentiation, we propose a simple model of a multiproduct duopoly. There are two segments of consumers, and each firm has a choice between offering a general purpose product and tailoring its offerings to one or both consumer segments. In contrast to Brander & Eaton's (1984) model, we let a firm choose both the number and type of products offered. To keep the model tractable, we assume, similar to Brander & Eaton (1984), that product attributes are given. We add two parameters that enable us to capture the tradeoff between general purpose and tailored products. One parameter represents fit, i.e., how much a consumer may gain from market segmentation, the other misfit, i.e., how much the consumer may lose. This parameterization enables us to extend the ideas behind models of spatial competition with endogenous transportation costs/gross benefits to multiproduct firms.

In the spirit of Gilbert & Matutes (1993), we allow consumers to have idiosyncratic preferences for firms. The stronger these brand preferences are, the softer the competition between firms is. Hence, even head-on competition generates positive profits, and a firm may find it in its best interest to duplicate the product offerings of its rival. Incorporating idiosyncratic brand preferences allows us to escape the unrealistic implication of models of spatial competition that firms will never compete head-on, say by both offering a general purpose product.[2]

[2] An alternative modeling strategy that may lead to head-on competition is to assume that firms choose quantities instead of prices. Indeed, De Fraja (1996) shows in a model of quality competition that quantity-setting firms always offer identical products. In a similar setup Johnson & Myatt

We identify four key determinants of market segmentation, namely the degree of fit, the degree of misfit, the intensity of competition, and the fixed cost of offering an additional product. We provide conditions under which in equilibrium (i) both firms forego the possibility of segmenting the market by offering a general purpose product; (ii) market segmentation occurs via niche firms; and (iii) market segmentation occurs via full-line firms. Our model is thus able to account for a variety of outcomes that we observe in the marketplace.

The remainder of this paper is organized as follows: Section 2 sets up the model. We characterize the equilibrium in Section 3 and summarize our results in Section 4. Section 5 concludes. The Appendix contains a number of robustness checks on our model.

2 Model

There are two firms, 1 and 2, and two segments of heterogeneous consumers, a and b. Each firm chooses between the following offerings:

- the general purpose product GP;
- product A which is targeted at segment a;
- product B which is targeted at segment b;
- or products A and B (denoted as AB).

That is, a firm that chooses to segment the market can be a niche firm and offer either A or B, or it can be a full-line firm and offer both A and B. In Appendix A, we allow a firm to offer any possible combination of A, B, and GP and show that our conclusions remain unchanged in this more general game.

(2003) study how an incumbent adjusts its product offerings to accommodate an entrant.

Firms compete in prices. We assume that if a firm offers more than one product, it charges the same price for all its products. This assumption simplifies the analysis considerably and is also satisfied or at least a good approximation to observed behavior in many product categories (see Shankar & Bolton (2004) and Draganska & Jain (2005) for examples). We relax this assumption in Appendix B and show that product-specific prices lead to minor changes in our results. The reason is that competition between firms limits their ability to exploit differences in consumers' willingness to pay by charging different prices for different products.

The marginal cost of production is $c \geq 0$. Turning to the fixed cost of production, we assume without loss of generality that the fixed cost of offering *one* product is zero and let $f \geq 0$ denote the fixed cost of offering an *additional* product. That is, we assume that manufacturing a larger number of products carries a greater fixed cost than manufacturing a smaller number of products. Moreover, we take the cost of the general purpose product to be the same as the cost of the tailored products. In contrast, von Ungern-Sternberg (1988) and Hendel & Neiva de Figueiredo (1998) assume that producing a general purpose product is more costly than producing a specific product, an *ad-hoc* assumption needed in their case to ensure an interior solution to a firm's profit maximization problem.

Consumers have unit demand. Consider a consumer in segment a who is looking at the products of firm 1. Her utility from the general purpose product at price p_1 is

$$v - p_1 + \epsilon,$$

where v is the gross benefit that the consumer derives form the general purpose product and ϵ is the consumer's relative idiosyncratic preference for firm 1 over firm 2 (see e.g. Fudenberg & Tirole 2000). This idiosyncratic preference may represent her taste for a brand name or company reputation. The consumer's utility from product A is

$$\overline{v} - p_1 + \epsilon$$

and her utility from product B is

$$\underline{v} - p_1 + \epsilon.$$

Similarly, the utility of a consumer in segment b who is looking at the products of firm 1 is $v - p_1 + \epsilon$ for the general purpose product, $\underline{v} - p_1 + \epsilon$ for product A, and $\overline{v} - p_1 + \epsilon$ for product B. The utility that a consumer derives from the products of firm 2 is given by analogous expressions with ϵ replaced by zero. We assume

$$\underline{v} < v < \overline{v}.$$

That is, a consumer in segment a (b) receives the highest gross benefit from product A (B), which is targeted at her own segment, and the lowest gross benefit from product B (A), which is targeted at the other segment. The gross benefit of the general purpose product is somewhere between the ones of products A and B. Hence, a consumer gains $\overline{v} - v$ from market segmentation if her preferred product is offered and loses $\underline{v} - v$ if her preferred product is not offered. In other words, $\overline{v} - v$ measures the *utility gain* from market segmentation due to the *increased fit* and $\underline{v} - v$ the *utility loss* due to the *increased misfit*.

We assume that the market is fully covered and that each segment has a unit mass of consumers. We further assume that $\epsilon \sim F$, where F is symmetric around zero, i.e., $F(\epsilon) = 1 - F(-\epsilon)$. Hence, if $\epsilon > 0$ ($\epsilon < 0$), then the consumer is biased in favor of firm 1 (firm 2).

Our parameters $\underline{v}$, v, and $\overline{v}$ can be viewed as combining transportation costs and gross benefits in traditional models of spatial competition. To see this, suppose that consumers in segments a and b are located at opposite ends of a Hotelling line of unit length and that the tailored products are located at the corresponding ends, whereas the general purpose product is located in the middle. Suppose further that gross benefits are different for the tailored products and the general purpose product. Then the utility (before price) that a consumer in segment a obtains from buying product A is $\nu^{A,B}$, where $\nu^{A,B}$ is the gross benefit of the tailored products. Note that in this case the transportation costs are zero. The utility that this consumer obtains from buying product B (GP) is $\nu^{A,B} - \tau$ ($\nu^{GP} - \frac{\tau}{2}$), where ν^{GP} is the gross benefit of the general purpose product and τ is the transportation cost per unit of distance. Hence, $\underline{v} = \nu^{A,B} - \tau$, $v = \nu^{GP} - \frac{\tau}{2}$, and $\overline{v} = \nu^{A,B}$. In this sense, by allowing a firm to choose the number and type of products offered, we endogenize both transportation costs and gross benefits.

We consider a two-stage game. The timing is as follows:

1. Firms simultaneously decide on their product offerings.

2. Price competition takes place.

We look for the subgame perfect equilibria (SPEs) of this two-stage game.

3 Equilibrium

We solve for SPEs by backwards induction.

Price competition. Suppose that firm 1 offers GP and firm 2 offers GP. Then a consumer in segment a buys from firm 1 if

$$v - p_1 + \epsilon \geq v - p_2 \Leftrightarrow \epsilon \geq p_1 - p_2$$

and a consumer in segment b buys from firm 1 if

$$v - p_1 + \epsilon \geq v - p_2 \Leftrightarrow \epsilon \geq p_1 - p_2.$$

Hence, total demand for firm 1 is $2(1 - F(p_1 - p_2)) = 2F(p_2 - p_1)$ and total demand for firm 2 is $2F(p_1 - p_2)$. Since firms offer the same product, consumers in both segments base their decisions solely on the difference in prices and their idiosyncratic preferences for firms.

Suppose next that firm 1 offers GP and firm 2 offers A. Then a consumer in segment a buys from firm 1 if

$$v - p_1 + \epsilon \geq \overline{v} - p_2 \Leftrightarrow \epsilon \geq \overline{v} - v + p_1 - p_2$$

and a consumer in segment b buys from firm 1 if

$$v - p_1 + \epsilon \geq \underline{v} - p_2 \Leftrightarrow \epsilon \geq \underline{v} - v + p_1 - p_2.$$

In this case, total demand for firm 1 is $1 - F(\overline{v} - v + p_1 - p_2) + 1 - F(\underline{v} - v + p_1 - p_2) = F(v - \overline{v} + p_2 - p_1) + F(v - \underline{v} + p_2 - p_1)$ and total demand for firm 2 is $F(\overline{v} - v + p_1 - p_2) + F(\underline{v} - v + p_1 - p_2)$. Since firms no longer offer the same product, consumers take into account the difference not only in prices but also in gross benefits when making their purchase decisions.

Finally suppose that firm 1 offers GP and firm 2 offers A and B. Then a consumer in segment a buys from firm 1 if

$$v - p_1 + \epsilon \geq \max\{\overline{v} - p_2, \underline{v} - p_2\} \Leftrightarrow \epsilon \geq \overline{v} - v + p_1 - p_2$$

and a consumer in segment b buys from firm 1 if

$$v - p_1 + \epsilon \geq \max\{\underline{v} - p_2, \overline{v} - p_2\} \Leftrightarrow \epsilon \geq \overline{v} - v + p_1 - p_2,$$

so that total demand for firm 1 is $2(1 - F(\overline{v} - v + p_1 - p_2)) = 2F(v - \overline{v} + p_2 - p_1)$ and total demand for firm 2 is $2F(\overline{v} - v + p_1 - p_2)$.

In general, total demand for firm 1 is of the form $F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1)$ and total demand for firm 2 is of the form $F(-\Delta_a + p_1 - p_2) + F(-\Delta_b + p_1 - p_2)$. $\Delta_a > 0$ implies that firm 1's product offerings better satisfy the needs of segment a than firm 2's product offerings. Similarly, Δ_b captures the match between firm 1's products and segment b's needs relative to firm 2's products. Table 1 lists (Δ_a, Δ_b) for all possible combinations of product offerings.

$1\backslash 2$	GP	A	B	AB
GP	$(0,0)$	$(v - \overline{v}, v - \underline{v})$	$(v - \underline{v}, v - \overline{v})$	$(v - \overline{v}, v - \overline{v})$
A	$(\overline{v} - v, \underline{v} - v)$	$(0,0)$	$(\overline{v} - \underline{v}, \underline{v} - \overline{v})$	$(0, \underline{v} - \overline{v})$
B	$(\underline{v} - v, \overline{v} - v)$	$(\underline{v} - \overline{v}, \overline{v} - \underline{v})$	$(0,0)$	$(\underline{v} - \overline{v}, 0)$
AB	$(\overline{v} - v, \overline{v} - v)$	$(0, \overline{v} - \underline{v})$	$(\overline{v} - \underline{v}, 0)$	$(0,0)$

Table 1: (Δ_a, Δ_b) for all possible combinations of product offerings. Firm 1 is row, firm 2 is column.

Given product offerings (Δ_a, Δ_b), gross profits (before fixed costs are subtracted) are

$$\pi_1(p_1, p_2) = \Big(F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1)\Big)(p_1 - c),$$

$$\pi_2(p_1,p_2) = \Big(F(-\Delta_a + p_1 - p_2) + F(-\Delta_b + p_1 - p_2)\Big)(p_2 - c),$$

and the first two derivatives of $\pi_1(p_1, p_2)$ with respect to p_1 are

$$\begin{aligned}
\frac{\partial}{\partial p_1}\pi_1(p_1,p_2) &= -\Big(F'(\Delta_a + p_2 - p_1) + F'(\Delta_b + p_2 - p_1)\Big)(p_1 - c) \\
&\quad + F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1), \\
\frac{\partial^2}{\partial p_1^2}\pi_1(p_1,p_2) &= \Big(F''(\Delta_a + p_2 - p_1) + F''(\Delta_b + p_2 - p_1)\Big)(p_1 - c) \\
&\quad -2\Big(F'(\Delta_a + p_2 - p_1) + F'(\Delta_b + p_2 - p_1)\Big).
\end{aligned}$$

Our next task is to characterize the Nash equilibrium (NE) of the pricing subgame. To this end, we assume that ϵ is logistically distributed with $F(\epsilon) = \frac{1}{1+e^{-\frac{\epsilon}{\beta}}}$, where $\beta > 0$ is proportional to the standard deviation of the idiosyncratic shock and measures how sensitive market shares are to differences in product offerings and prices between the two firms. Indeed, as β grows, consumers are more prone to buy from their favorite firm irrespective of product offerings and prices. We thus say that consumers have strong (weak) *brand preferences* if β is high (low). Consequently, β is also a measure of the *intensity of competition*: If β is high, competition is soft, and if β is low, competition is fierce.

Using the properties of the logistic distribution, we have $F'(\epsilon) = \frac{1}{\beta}F(\epsilon)(1 - F(\epsilon))$ and $F''(\epsilon) = \frac{1}{\beta^2}F(\epsilon)(1 - F(\epsilon))(1 - 2F(\epsilon))$. In the special case of $\Delta_a = \Delta_b$, the product offerings of, say, firm 1 are equally suited to both segments of consumers. From the firm's perspective, it is thus as if there were just one large segment. It is now straightforward to show $\pi_1(p_1, p_2)$ is strictly quasiconcave in p_1, so that results by Caplin & Nalebuff (1991) imply the existence of a unique NE of the pricing subgame (see also Chapters 6 and 7 of Anderson, de Palma & Thisse (1992) and Mizuno (2003)). While no analytic solution is available, the NE can be computed by numerically solving the system of FOCs given by $\frac{\partial \pi_1}{\partial p_1}(p_1, p_2) = 0$ and $\frac{\partial \pi_2}{\partial p_2}(p_1, p_2) = 0$.[3]

Unfortunately, these results no longer apply if $\Delta_a \neq \Delta_b$, and there may in fact not be a NE in pure strategies. To see this, suppose that firm 1's product offerings satisfy the needs of segment a well, $\Delta_a \gg 0$, and the needs of segment b poorly, $\Delta_b \ll 0$. Then the firm faces a choice between exploiting the consumers in segment a by setting a high price and setting a low price in segment b to be competitive. This may give rise to a discontinuity in the firm's best reply function and ultimately lead to the nonexistence of a NE in pure strategies. By verifying that neither firm has a profitable unilateral deviation, however, it is easy to ensure that a numerical solution to the system of FOCs constitutes a NE of the pricing subgame.

Figure 1 depicts the equilibrium market shares of firm 1 in segments a and b, its price, and its profit as a function of (Δ_a, Δ_b). The equilibrium price of firm 2 is given by $p_2^*(\Delta_a, \Delta_b) = p_1^*(-\Delta_a, -\Delta_b)$, its equilibrium profit by $\pi_2^*(\Delta_a, \Delta_b) = \pi_1^*(-\Delta_a, -\Delta_b)$. Figure 1 is based on $\beta = 0.5$ and $c = 0$. These parameter values imply rather fierce competition, with the elasticity of firm 1's demand with respect to firm 2's price ranging from 0.32 at $(-2, -2)$ to 3.16 at $(2, 2)$. As can be seen, for some $(\Delta_a, \Delta_b) \in [-2, 2]^2$, there is no NE of the pricing subgame. One intuitively suspects that existence becomes less problematic as competition becomes less intense, and our computations readily confirm this. For example, if we increase β from 0.5 to 1, then there is a NE for all product offerings.

Contrary to what one may expect, $p_1^*(\Delta_a, \Delta_b)$ and $\pi_1^*(\Delta_a, \Delta_b)$ are not necessarily increasing in Δ_a and Δ_b. That is, a better match between a firm's products and consumer's needs does not directly translate into higher prices and profits. Consider

[3]We were unable to obtain analytic solutions for a variety of alternative assumptions on the distribution of brand preferences. The uniform distribution, for example, turns out to be intractable because it requires a case-by-case analysis. To ensure the robustness of our results, we have recomputed the NE of the pricing subgame using a normal and a double exponential (Laplace) distribution. For details see the Online Appendix at the *Journal's* website.

again a situation in which firm 1's product offerings satisfy the needs of segment a well, $\Delta_a \gg 0$, and the needs of segment b poorly, $\Delta_b \ll 0$. Then an increase in Δ_b leads *ceteris paribus* to a large increase in firm 1's share of segment b. Firm 2 reacts to this large decrease in its share of segment b by dropping its price sharply, which in turn forces firm 1 to drop its price. Overall, the total effect on firm 1's profit is negative. On the other hand, consider a situation in which firm 1's product offerings satisfy the needs of both segments well, $\Delta_a \gg 0$ and $\Delta_b \gg 0$. Then an increase in Δ_b leads *ceteris paribus* to a small increase in firm 1's share of segment b. Firm 2 reacts to this small decrease in its share of segment b by dropping its price marginally. This leaves firm 1 room for a price increase. Overall, the total effect on firm 1's profit is positive.

More generally, as the following proposition shows, $p_1^*(\Delta_a, \Delta_b)$ and $\pi_1^*(\Delta_a, \Delta_b)$ are increasing in Δ_a and Δ_b provided that $\Delta_a \approx \Delta_b$, i.e., provided that the product offerings of both firms are similarly suited to both segments of consumers. It follows immediately that $p_2^*(\Delta_a, \Delta_b)$ and $\pi_2^*(\Delta_a, \Delta_b)$ are decreasing in Δ_a and Δ_b provided that $\Delta_a \approx \Delta_b$.

Proposition 1 *Suppose* $\Delta_a \approx \Delta_b$. *Then* $\frac{\partial p_1^*}{\partial \Delta_a} > 0$, $\frac{\partial p_1^*}{\partial \Delta_b} > 0$, $\frac{\partial \pi_1^*}{\partial \Delta_a} > 0$, *and* $\frac{\partial \pi_1^*}{\partial \Delta_b} > 0$.

Proof. Totally differentiate the system of FOCs. Use the fact that $\Delta_a = \Delta_b$ implies $F(\Delta_a + p_2^* - p_1^*) = F(\Delta_b + p_2^* - p_1^*)$ along with $F(\epsilon) = 1 - F(-\epsilon)$ to obtain

$$\frac{\partial p_1^*}{\partial \Delta_a} = \frac{\partial p_1^*}{\partial \Delta_b} = \frac{F(\Delta_a + p_2^* - p_1^*)^2}{2(1 - F(\Delta_a + p_2^* - p_1^*) + F(\Delta_a + p_2^* - p_1^*)^2)} > 0,$$
$$\frac{\partial p_2^*}{\partial \Delta_a} = \frac{\partial p_2^*}{\partial \Delta_b} = \frac{-(1 - F(\Delta_a + p_2^* - p_1^*))^2}{2(1 - F(\Delta_a + p_2^* - p_1^*) + F(\Delta_a + p_2^* - p_1^*)^2)} < 0,$$

where we rewrite the FOCs as $\frac{p_1^* - c}{\beta} = \frac{1}{1 - F(\Delta_a + p_2^* - p_1^*)}$ and $\frac{p_2^* - c}{\beta} = \frac{1}{F(\Delta_a + p_2^* - p_1^*)}$ to substitute for the price-cost margins. It follows that

$$\frac{\partial \pi_1^*}{\partial \Delta_a} = \frac{\partial \pi_1^*}{\partial \Delta_b} = \frac{F(\Delta_a + p_2^* - p_1^*)^2}{1 - F(\Delta_a + p_2^* - p_1^*) + F(\Delta_a + p_2^* - p_1^*)^2} > 0.$$

The proof is completed by noting that continuity implies that the above carries over to a neighborhood of $\Delta_a = \Delta_b$. ■

Product offerings. The stage game is finite. Provided the existence of a NE in all relevant pricing subgames, the overall game has at least one SPE (possibly in mixed strategies). Our goal is to characterize the set of SPEs and to describe how it changes as the parameters $\underline{v}$, v, $\overline{v}$, β, c, and f change.

The simplicity of our model allows us to reduce the number of parameters, thereby aiding the subsequent analysis. First, observe from Table 1 that only differences between $\overline{v}$, v, and $\underline{v}$ matter for determining total demand and hence firms' profits. We can therefore set $v = 0$ without loss of generality. This implies that $\overline{v} > 0$ ($\underline{v} < 0$) measures the gross benefit a consumer gains (loses) from buying a product targeted at her own segment (the other segment) instead of the general purpose product. In other words, $\overline{v}$ ($\underline{v}$) measures the utility gain (loss) due to the *increased fit (misfit)*, and we henceforth interpret $\overline{v}$ ($|\underline{v}|$) as a measure of the degree of fit (misfit). Note next that the marginal cost of production determines the extent of the transfer from consumers to firms but has no impact on firms' profits in equilibrium. We thus set $c = 0$. This leaves $\underline{v}$, $\overline{v}$, β, and f as the parameters of interest. Firm 1's payoffs are listed in Table 2, firm 2's payoffs are analogous. Note that we have slightly changed our notation to emphasize that these payoffs also depend on the *intensity of competition* β. Recall also that f is the *fixed cost of offering an additional product.*

As an inspection of Table 2 suggests, it is in general impossible to determine the

$1\backslash 2$	GP	A	B	AB
GP	$\pi_1^*(0,0;\beta)$	$\pi_1^*(-\overline{v},-\underline{v};\beta)$	$\pi_1^*(-\underline{v},-\overline{v};\beta)$	$\pi_1^*(-\overline{v},-\overline{v};\beta)$
A	$\pi_1^*(\overline{v},\underline{v};\beta)$	$\pi_1^*(0,0;\beta)$	$\pi_1^*(\overline{v}-\underline{v},\underline{v}-\overline{v};\beta)$	$\pi_1^*(0,\underline{v}-\overline{v};\beta)$
B	$\pi_1^*(\underline{v},\overline{v};\beta)$	$\pi_1^*(\underline{v}-\overline{v},\overline{v}-\underline{v};\beta)$	$\pi_1^*(0,0;\beta)$	$\pi_1^*(\underline{v}-\overline{v},0;\beta)$
AB	$\pi_1^*(\overline{v},\overline{v};\beta)-f$	$\pi_1^*(0,\overline{v}-\underline{v};\beta)-f$	$\pi_1^*(\overline{v}-\underline{v},0;\beta)-f$	$\pi_1^*(0,0;\beta)-f$

Table 2: Payoffs to firm 1 for all possible combinations of product offerings. Firm 1 is row, firm 2 is column.

set of SPEs without rather detailed information about the properties of $\pi_1^*(\Delta_a, \Delta_b; \beta)$. Nevertheless, we can show that the unique SPE is market segmentation with full-line firms if the degree of fit as well as the degree of misfit are sufficiently low and if the fixed cost of offering an additional product is sufficiently small.

Proposition 2 *Suppose $\underline{v} \approx 0$, $\overline{v} \approx 0$, and $f \approx 0$. Then the unique SPE is market segmentation with full-line firms.*

Proof. Taken together $\underline{v} \approx 0$ and $\overline{v} \approx 0$ ensure that $\Delta_a \approx \Delta_b$ for all possible combinations of product offerings in Table 2. Hence, Proposition 1 applies and we have $\frac{\partial \pi_1^*}{\partial \Delta_a} > 0$ and $\frac{\partial \pi_1^*}{\partial \Delta_b} > 0$. In light of $\underline{v} < 0 < \overline{v}$ firm 1 is seen to maximize its gross profit by offering both A and B irrespective of firm 2's product offerings. Given that the fixed cost of offering an additional product f is sufficiently small, offering a full line is thus the dominant strategy, and the claim follows. ■

To better understand what is going on, suppose firm 2 offers both A and B. Then there is no point for firm 1 to offer GP because both segments of consumers can buy their favorite product from firm 2. Moreover, unlike models of spatial competition such as Martinez-Giralt & Neven (1988), consumers have idiosyncratic preferences for brands in our setup. Hence, head-on competition can still generate positive profits. While these profits are small, so is the fixed cost of offering an additional product, and it is best for firm 1 to also offer both A and B. Indeed, offering a full line is the dominant strategy for both firms. It follows that the SPE is unique.

Computation. Since it is impossible to get closed-form solutions for the pricing subgame, we employ numerical methods and use of the fact that, in two-player games, the set of equilibria with a given support is convex (possibly empty) (Mangasarian 1964).[4] Hence, it is feasible to enumerate all equilibria using the following algorithm: For each possible support, we check for an equilibrium. If there is one, it is either unique or we can find a finite set of extreme points, the convex hull of which represents the set of equilibria for that support.[5] A formal description of Mangasarian's (1964) algorithm can be found in Section 6.3.1 of McKelvey & McLennan (1996). It has been implemented in Gambit 0.96.3. The payoffs themselves are computed in Matlab 5.3. We use the c05nbf routine of the NAG toolbox, a Newton method, to solve the system of FOCs.

Parameterization. Given a point (β, f) we compute the set of equilibria over a grid of 25^2 equidistant points $(\underline{v}, \overline{v})$ in $[-1, 0) \times (0, 1]$, implying $(\Delta_a, \Delta_b) \in [-2, 2]^2$. To explore how the intensity of competition affects the set of equilibria, we choose a fairly wide range of values with $\beta \in \{0.5, 1, 2, 4\}$. The corresponding maximal elasticity of firm 1's demand with respect to firm 2's price decreases from 3.16 over 1.89 and 1.39 to 1.18 and the corresponding minimal elasticity increases from 0.32 over 0.53 and 0.72 to 0.85. Next observe that, given β, the set of equilibria remains the same for a sufficiently large fixed cost f. Because profits from price competition are nonnegative, offering both products eventually becomes a dominated strategy. This provides us with an upper bound for f. To find it, we start with $f = 0$ and

[4] For games with more than two players, the set of equilibria with a given support need no longer be convex (or even connected) (McKelvey & McLennan 1996, p. 132).

[5] Since almost all finite games have a finite (and odd) number of equilibria (Wilson 1971), we content ourselves with computing the extreme points of the set of the equilibria.

increase f in increments of 0.1 until the set of equilibria remains the same.[6]

4 Results

Let s_n denote a (mixed) strategy for firm n, i.e., s_n is a probability distribution over the feasible actions GP, A, B, and AB. In general, there are multiple equilibria for given values of $\underline{v}$, $\overline{v}$, β, and f. This multiplicity partly reflects the symmetry of the game. In particular, if (s_1^*, s_2^*) is an equilibrium, then (s_2^*, s_1^*) is also an equilibrium. Since (s_1^*, s_2^*) and (s_2^*, s_1^*) are the same up to a relabelling of firms, we do not distinguish between them in what follows.

Even after dismissing such multiplicity, there remains a considerable number of equilibria. For example, if $\underline{v} = -0.96$, $\overline{v} = 0.04$, $\beta = 0.5$, and $f = 0.05$, then there are multiple equilibria in pure strategies: one where one firm offers A and the other offers B, and one where both firms offer GP. In addition, there are multiple equilibria in mixed strategies: one where both firms randomize between offering A and B with probabilities 0.5 and 0.5; one where one firm randomizes between offering GP and A with probabilities 0.99 and 0.01, and the other firm randomizes between offering GP and B with probabilities 0.99 and 0.01; and one where both firms randomize between offering GP, A, and B with probabilities 0.93, 0.04, and 0.04.[7]

Pareto-undominated equilibria. In what follows we focus on Pareto-undominated equilibria because, given a Pareto-dominated equilibrium, it is in the interest of both firms to coordinate on a different equilibrium. In the above example, the general-

[6] If $\beta = 0.5$, the set of equilibria remains the same for $f \geq 0.2$; if $\beta = 1$, the set of equilibria remains the same for $f \geq 0.3$; if $\beta = 2$, the set of equilibria remains the same for $f \geq 0.5$; and if $\beta = 4$, the set of equilibria remains the same for $f \geq 0.6$. The set of Pareto-undominated equilibria remains the same for $f \geq 0.1$, $f \geq 0.1$, $f \geq 0.2$, and $f \geq 0.4$, respectively.

[7] This multiplicity is reminiscent of Brander & Eaton (1984), where both market segmentation and market interlacing are an equilibrium.

purpose equilibrium as well as all mixed-strategy equilibria are Pareto-dominated by the segmentation equilibrium with niche firms. In fact, the gains from coordinating on the Pareto-undominated equilibrium are substantial: Each firm's profit increases by between 41% and 138%. This presumably makes market segmentation with niche firms the most likely outcome.

The computations lead to a unique Pareto-undominated equilibrium in pure strategies in all cases. Depending on the values of $\underline{v}$, $\overline{v}$, β, and f, the Pareto-undominated equilibrium involves either both firms offering the general purpose product, (GP, GP), market segmentation with niche firms, (A, B) (or (B, A)), or market segmentation with full-line firms, (AB, AB). Figures 2-5 illustrate the results for $\beta = 0.5$, $f \in \{0, 0.05\}$, and $f \geq 0.1$; $\beta = 1$, $f \in \{0, 0.05\}$, and $f \geq 0.1$; $\beta = 2$, $f \in \{0, 0.05, 0.1\}$, and $f \geq 0.2$; as well as $\beta = 4$, $f \in \{0, 0.05, 0.1, 0.2, 0.3\}$, and $f \geq 0.4$. In each figure we mark the point $(\underline{v}, \overline{v})$ for which a particular equilibrium occurs as follows: (GP, GP) is denoted by a blue star, (A, B) (or (B, A)) by a red x-mark, and (AB, AB) by a green plus. $\underline{v}$ is denoted as vl, $\overline{v}$ as vh. Note that Figure 2 is blank for some points $(\underline{v}, \overline{v})$. This indicates that a NE does not exist in all relevant pricing subgames.

General purpose products. If the fixed cost of offering an additional product f is zero, then there is no general-purpose equilibrium. Instead, the market is segmented. If the fixed cost is positive, then (GP, GP) may be an equilibrium. The tendency towards general purpose products is reinforced by a higher fixed cost.

Both firms forego the possibility of segmenting the market by offering the general purpose product if the degree of fit is not only low (i.e., $\overline{v}$ low) but also lower than the degree of misfit (i.e., $\overline{v} < |\underline{v}|$). In other words, for the general-purpose equilibrium to be viable, the utility a consumer gains from buying a product targeted at her own segment has to be less than the utility she loses from buying a product targeted at the other segment. Otherwise a firm would be able to profit by segmenting the market and capturing part of the resulting gain through charging a high price.

Strong brand preferences β facilitate general purpose products. This is in line with our intuition: Since the intensity of competition is low when β is high, avoiding head-on competition through market segmentation becomes less of a concern for firms.

Market segmentation with niche firms. A high fixed cost of offering an additional product f leads to niche firms, whereas strong brand preferences β inhibit niche firms. In fact, (A, B) (or (B, A)) cannot be an equilibrium if the fixed cost is zero and brand preferences are sufficiently high (see $\beta = 4$ and $f = 0$ in Figure 5).

(A, B) (or (B, A)) may occur in two quite distinct situations. If β is low, it occurs when the degree of fit and misfit is *high* (i.e., $|\underline{v}|$ and $\overline{v}$ large; see Figure 2). In this situation, a consumer gains considerably from being able to buy a product that satisfies her needs. By segmenting the market rather than offering a general purpose product, a firm can partake in this large gain by charging a higher price. However, the consumer loses a lot because she cannot buy a product that satisfies her needs. So why then do firms not offer a full line of products? To see what is going on, consider $\underline{v} = -0.64$, $\overline{v} = 0.64$, $\beta = 0.5$, and $f = 0.05$. If firm 1 were to deviate by offering B in addition to A while firm 2 continues to offer B, then firm 2 would drop its price from $p_2^*(-1.28, 1.28; 0.5) = 3.75$ to $p_2^*(-1.28, 0; 0.5) = 1.26$. This in turn would force firm 1 to cut its price to $p_1^*(-1.28, 0; 0.5) = 1.69$, causing its profit to fall by 50%. More generally, it is precisely because the difference between the utility gain and the utility loss from market segmentation is significant that a firm punishes any infringement on 'its' segment by aggressively pushing prices, and hence profits, down. This keeps a segmentation equilibrium with niche firms in place.

In contrast, (A, B) (or (B, A)) occurs when the degree of fit and the degree of misfit are both *low* (i.e., $|\underline{v}|$ and $\overline{v}$ are low) provided that β is high (see Figure 5). This may seem surprising at first glance. Recall from Table 2 that firm 1's profit is $\pi_1^*(0, 0; \beta)$ under head-on competition and $\pi_1^*(\overline{v} - \underline{v}, \underline{v} - \overline{v}; \beta)$ under (A, B). Because $\underline{v} \approx 0$ and $\overline{v} \approx 0$ imply $\pi_1^*(\overline{v} - \underline{v}, \underline{v} - \overline{v}; \beta) \approx \pi_1^*(0, 0; \beta)$, there is little to be gained from segmenting the market in this situation. Yet, as long as the gain is not zero, niche firms arise.

Lastly, combining the above considerations, if β and, in addition, f is intermediate, (A, B) (or (B, A)) occurs when the degree of fit and the degree of misfit are both high and when they are both low (see $\beta = 1$ and $f = 0.05$ in Figure 3 and $\beta = 2$ and $f \in \{0.05, 0.1\}$ in Figure 4).

Market segmentation with full-line firms. In line with Proposition 2, market segmentation occurs via full-line firms if the degree of fit and misfit is sufficiently low (i.e., $\overline{v} \approx 0$ and $\underline{v} \approx 0$) and if the fixed cost of offering an additional product is sufficiently small (i.e., $f \approx 0$, see top left panels of Figures 2-5). A high fixed cost of offering an additional product f, by contrast, inhibits full-line firms. In fact, (AB, AB) cannot be an equilibrium if the fixed cost is sufficiently high. Strong brand preferences β make head-on competition more profitable and thus facilitate the creation of full-line firms.

Market segmentation with full-line firms takes up the remaining part of $(\underline{v}, \overline{v})$-space. In particular, if β and f are intermediate, (AB, AB) occurs when either the degree of fit or misfit is high, while the other is low (see again Figures 3 and 4).

Discussion. Table 3 summarizes the four key determinants of market segmentation. It lists conditions under which in equilibrium both firms forego the possibility of segmenting the market by offering a general purpose product, (GP, GP); market segmentation occurs via niche firms, (A, B) (or (B, A)); or market segmentation occurs via full-line firms, (AB, AB).

	general purpose products	market segmentation with niche firms	market segmentation with full-line firms
competition	soft	fierce	soft
fixed cost	high	high	low
degree of fit degree of misfit	degree of fit lower than degree of misfit	either both high or both low	one high, the other low

Table 3: Key determinants of market segmentation.

A higher intensity of competition makes head-on competition less profitable and thus gives firms an incentive to specialize. If the intensity of competition is low, on the other hand, a firm's best interest may be to duplicate the product offerings of its rival. If the fixed cost of offering an additional product is low, firms compete with a full line of products, whereas they compete with general purpose products if the fixed cost is high.

(GP, GP) and (AB, AB) leads to head-on competition and (AB, AB), in addition, to multiproduct firms. This result stands in marked contrast to models of spatial competition, where price competition gives rise to Hotelling's principle of maximum differentiation and, *de facto*, prevents multiproduct firms. Hence, by not allowing consumers to have preferences over brands the existing models of multiproduct firms (e.g., Martinez-Giralt & Neven 1988) miss an important part of the story.

What role does the tradeoff between fit and misfit inherent in market segmentation play? Both firms forego the possibility of segmenting the market by offering the general purpose product if the degree of fit is not only low but also lower than the degree of misfit. Otherwise, the market is segmented in equilibrium. Niche firms result either when firms have a lot to gain from specializing or when they have too little to gain. The latter situation arises if the degree of fit and misfit is low, while the former occurs if it is high. A firm is prevented from offering a full line of products because its rival responds to such an infringement by aggressively pushing prices, and hence profits, down. If either the degree of fit or misfit is high while the other is low, full-line firms result.

It is well-worth comparing our results to those of Gilbert & Matutes (1993). They consider a model of quality competition and, just like we do, allow consumers to have idiosyncratic preferences for firms. In their model, firms in general adopt a full-line strategy because markups on different products are identical and the fixed cost of offering an additional product is assumed to be zero. This leads to a prisoners' dilemma since in equilibrium both firms offer both products although this leaves them no better off than they would have been with a restricted product line. Our findings indicate that whether or not firms pursue a full-line strategy hinges on the degree of fit and misfit even in the special case of $f = 0$. Furthermore, Gilbert & Matutes (1993) argue that firms can avoid being caught in a prisoner's dilemma provided they can credibly commit to not offering a product. In our model, by contrast, firms do not necessarily adopt a full-line strategy despite the absence of such commitments.

5 Conclusions

This paper contributes to the existing literature by considering the tradeoff between the positive and negative aspects of market segmentation, fit and misfit, that multiproduct firms face when deciding on the number and type of products to offer. We show that firms' market segmentation strategies are influenced by four factors, namely the intensity of competition, the fixed cost of offering an additional product, the degree of fit, and the degree of misfit.

Compared to previous work, which either concludes that there can be no multiproduct firms, or that firms have a strong incentive to produce general purpose products and the degree of product differentiation in the marketplace is quite low, our model accounts for a variety of outcomes consistent with the market segmentation strategies that different firms in different industries pursue.

Future work should look into generalizing the model presented here. In its present form, our model implies that gross profits (before fixed costs are subtracted) are the same when both firms offer the general purpose product and when they both offer full lines. This may be relaxed by allowing consumers to opt for a no-purchase alternative. Next, a firm has no incentive to target one segment of consumers and offer the general purpose product because both segments are the same size and the fixed cost of production depends on the number - not type - of products. If either of these assumptions is relaxed, then a firm may choose to partially segment the market. More generally, the model should be extended to more than two firms, two segments of consumers, and a correspondingly larger set of possible product offerings.

Appendix A

In what follows, we allow a firm to offer any possible combination of A, B, and GP. In particular, each firm chooses between the following offerings:

- the general purpose product GP;
- product A which is targeted at segment a;
- product B which is targeted at segment b;
- products A and B (denoted as AB);
- the general purpose product GP and product A (denoted as GPA);

- the general purpose product GP and product B (denoted as GPB);

- or the general purpose product GP and products A and B (denoted as $GPAB$).

Firm 1's payoffs are listed in Table 4, firm 2's payoffs are analogous. We show that our conclusions from the main text remain unchanged in this more general game.

Consider first GPA and GPB. These product offerings cannot be part of a SPE if the degree of fit as well as the degree of misfit are sufficiently low.

Proposition 3 *Suppose* $\underline{v} \approx 0$ *and* $\overline{v} \approx 0$. *Then* GPA *and* GPB *are dominated by* AB.

Proof. Taken together $\underline{v} \approx 0$ and $\overline{v} \approx 0$ ensure that $\Delta_a \approx \Delta_b$ for all possible combinations of product offerings. Hence, Proposition 1 applies and we have $\frac{\partial \pi_1^*}{\partial \Delta_a} > 0$ and $\frac{\partial \pi_1^*}{\partial \Delta_b} > 0$. In light of $\underline{v} < 0 < \overline{v}$, firm 1 obtains a higher payoff by offering both A and B instead of both GP and A or both GP and B irrespective of firm 2's product offerings. Hence, the claim follows. ■

Our computations reveal that the above result continues to hold even if $\underline{v} \ll 0$ or $\overline{v} \gg 0$. That is, GPA and GPB are never part of a SPE, and analyzing a game that rules out these product offerings is without loss of generality.

Next consider $GPAB$. This product offering cannot be part of a SPE if the fixed cost of offering an additional product is positive.

Proposition 4 *Suppose* $f > 0$. *Then* $GPAB$ *is dominated by* AB.

Proof. Follows immediately from an inspection of Table 4. ■

On the other hand, if $f = 0$, then $GPAB$ yields the same payoff as AB. In this knife-edge case, a firm is indifferent between offering a full line and offering the general purpose product in addition to a full line. Given a SPE of the game analyzed in the main text that puts positive probability on AB, there is thus an additional

$1\backslash 2$	GP	A	B	AB	GPA	GPB	$GPAB$
GP	$\pi_1^*(0,0;\beta)$	$\pi_1^*(-\overline{v},-\underline{v};\beta)$	$\pi_1^*(-\underline{v},-\overline{v};\beta)$	$\pi_1^*(-\overline{v},-\overline{v};\beta)$	$\pi_1^*(-\overline{v},0;\beta)$	$\pi_1^*(0,-\overline{v};\beta)$	$\pi_1^*(-\overline{v},-\overline{v};\beta)$
A	$\pi_1^*(\overline{v},\underline{v};\beta)$	$\pi_1^*(0,0;\beta)$	$\pi_1^*(\overline{v}-\underline{v},\underline{v}-\overline{v};\beta)$	$\pi_1^*(0,\underline{v}-\overline{v};\beta)$	$\pi_1^*(0,\underline{v};\beta)$	$\pi_1^*(\overline{v},\underline{v}-\overline{v};\beta)$	$\pi_1^*(0,\underline{v}-\overline{v};\beta)$
B	$\pi_1^*(\underline{v},\overline{v};\beta)$	$\pi_1^*(\underline{v}-\overline{v},\overline{v}-\underline{v};\beta)$	$\pi_1^*(0,0;\beta)$	$\pi_1^*(\underline{v}-\overline{v},0;\beta)$	$\pi_1^*(\underline{v},\overline{v};\beta)$	$\pi_1^*(\underline{v},0;\beta)$	$\pi_1^*(\underline{v}-\overline{v},0;\beta)$
AB	$\pi_1^*(\overline{v},\overline{v};\beta)-f$	$\pi_1^*(0,\overline{v}-\underline{v};\beta)-f$	$\pi_1^*(\overline{v}-\underline{v},0;\beta)-f$	$\pi_1^*(0,0;\beta)-f$	$\pi_1^*(0,\overline{v};\beta)-f$	$\pi_1^*(\overline{v},0;\beta)-f$	$\pi_1^*(0,0;\beta)-f$
GPA	$\pi_1^*(\overline{v},0;\beta)-f$	$\pi_1^*(0,-\underline{v};\beta)-f$	$\pi_1^*(\overline{v}-\underline{v},-\overline{v};\beta)-f$	$\pi_1^*(0,-\overline{v};\beta)-f$	$\pi_1^*(0,0;\beta)-f$	$\pi_1^*(\overline{v},-\overline{v};\beta)-f$	$\pi_1^*(0,-\overline{v};\beta)-f$
GPB	$\pi_1^*(0,\overline{v};\beta)-f$	$\pi_1^*(-\overline{v},\overline{v}-\underline{v};\beta)-f$	$\pi_1^*(-\underline{v},0;\beta)-f$	$\pi_1^*(-\overline{v},0;\beta)-f$	$\pi_1^*(-\overline{v},\overline{v};\beta)-f$	$\pi_1^*(0,0;\beta)-f$	$\pi_1^*(-\overline{v},0;\beta)-f$
$GPAB$	$\pi_1^*(\overline{v},\overline{v};\beta)-2f$	$\pi_1^*(0,\overline{v}-\underline{v};\beta)-2f$	$\pi_1^*(\overline{v}-\underline{v},0;\beta)-2f$	$\pi_1^*(0,0;\beta)-2f$	$\pi_1^*(0,\overline{v};\beta)-2f$	$\pi_1^*(\overline{v},0;\beta)-2f$	$\pi_1^*(0,0;\beta)-2f$

Table 4: Payoffs to firm 1 for all possible combinations of product offerings. Firm 1 is row, firm 2 is column.

SPE of the more general game that puts the same probability on $GPAB$ instead of AB. (In addition, by taking convex combinations of these SPEs, one can construct a continuum of payoff-equivalent SPEs.) Note however that no consumer buys the general purpose product if it is offered in addition to a full line. Hence, a SPE that involves $GPAB$ is arguably indistinguishable from a SPE that involves AB.

Appendix B

In the main text, we assume that a firm charges the same price for all its products. This greatly simplifies the analysis by ensuring that the gross profit (before fixed costs are subtracted) of, say, firm 1, can be written as

$$\pi_1(p_1, p_2) = \Big(F(\Delta_a + p_2 - p_1) + F(\Delta_b + p_2 - p_1)\Big)(p_1 - c)$$

irrespective of the prices charged. Because of symmetry our assumption does not constrain firms' pricing behavior unless one firm offers both A and B and the other either A or B or unless both firms offer both A and B. In what follows, we relax this assumption and show that product-specific prices lead to minor changes in our results.

To see how product-specific prices complicate the analysis, suppose first that firm 1 offers A at price p_1^A and B at price p_1^B and firm 2 offers A. Then a consumer in segment a buys from firm 1 if

$$\max\left\{\overline{v} - p_1^A, \underline{v} - p_1^B\right\} + \epsilon \geq \overline{v} - p_2 \Leftrightarrow \epsilon \geq \begin{cases} p_1^A - p_2 & \text{if} \quad p_1^A - p_1^B \leq \overline{v} - \underline{v}, \\ \overline{v} - \underline{v} + p_1^B - p_2 & \text{if} \quad p_1^A - p_1^B > \overline{v} - \underline{v} \end{cases}$$

and a consumer in segment b buys from firm 1 if

$$\max\left\{\underline{v}-p_1^A,\overline{v}-p_1^B\right\}+\epsilon \geq \underline{v}-p_2 \Leftrightarrow \epsilon \geq \begin{cases} p_1^A-p_2 & \text{if} \quad p_1^A-p_1^B \leq \underline{v}-\overline{v}, \\ \underline{v}-\overline{v}+p_1^B-p_2 & \text{if} \quad p_1^A-p_1^B > \underline{v}-\overline{v}. \end{cases}$$

Hence, letting $\Delta_a = \underline{v}-\overline{v}$ and $\Delta_b = \overline{v}-\underline{v}$, firm 1's gross profit becomes

$$\pi_1(p_1^A,p_1^B,p_2) \quad = \quad \begin{cases} \Big(F(p_2-p_1^A)+F(p_2-p_1^A)\Big)(p_1^A-c) & \text{if} \quad p_1^A-p_1^B \leq \Delta_a, \\ F(p_2-p_1^A)(p_1^A-c)+F(\Delta_b+p_2-p_1^B)(p_1^B-c) & \text{if} \quad \Delta_a < p_1^A-p_1^B \leq \Delta_b, \\ \Big(F(\Delta_a+p_2-p_1^B)+F(\Delta_b+p_2-p_1^B)\Big)(p_1^B-c) & \text{if} \quad p_1^A-p_1^B > \Delta_b. \end{cases}$$

Firm 2's gross profit is analogous, as are the gross profits in the remaining situations in which one firm offers both A and B and the other either A or B.

Next suppose that firm 1 offers A at price p_1^A and B at price p_1^B and firm 2 offers A at price p_2^A and B at price p_2^B. Then a consumer in segment a buys from firm 1 if

$$\begin{aligned} &\max\left\{\overline{v}-p_1^A,\underline{v}-p_1^B\right\}+\epsilon \geq \max\left\{\overline{v}-p_2^A,\underline{v}-p_2^B\right\} \\ \Leftrightarrow \quad &\epsilon \geq \begin{cases} p_1^A-p_2^A & \text{if} \quad p_1^A-p_1^B \leq \overline{v}-\underline{v} \wedge p_2^A-p_2^B \leq \overline{v}-\underline{v}, \\ \underline{v}-\overline{v}+p_1^A-p_2^B & \text{if} \quad p_1^A-p_1^B \leq \overline{v}-\underline{v} \wedge p_2^A-p_2^B > \overline{v}-\underline{v}, \\ \overline{v}-\underline{v}+p_1^B-p_2^A & \text{if} \quad p_1^A-p_1^B > \overline{v}-\underline{v} \wedge p_2^A-p_2^B \leq \overline{v}-\underline{v}, \\ p_1^B-p_2^B & \text{if} \quad p_1^A-p_1^B > \overline{v}-\underline{v} \wedge p_2^A-p_2^B > \overline{v}-\underline{v} \end{cases} \end{aligned}$$

and a consumer in segment b buys from firm 1 if

$$\begin{aligned} &\max\left\{\underline{v}-p_1^A,\overline{v}-p_1^B\right\}+\epsilon \geq \max\left\{\underline{v}-p_2^A,\overline{v}-p_2^B\right\} \\ \Leftrightarrow \quad &\epsilon \geq \begin{cases} p_1^A-p_2^A & \text{if} \quad p_1^A-p_1^B \leq \underline{v}-\overline{v} \wedge p_2^A-p_2^B \leq \underline{v}-\overline{v}, \\ \overline{v}-\underline{v}+p_1^A-p_2^B & \text{if} \quad p_1^A-p_1^B \leq \underline{v}-\overline{v} \wedge p_2^A-p_2^B > \underline{v}-\overline{v}, \\ \underline{v}-\overline{v}+p_1^B-p_2^A & \text{if} \quad p_1^A-p_1^B > \underline{v}-\overline{v} \wedge p_2^A-p_2^B \leq \underline{v}-\overline{v}, \\ p_1^B-p_2^B & \text{if} \quad p_1^A-p_1^B > \underline{v}-\overline{v} \wedge p_2^A-p_2^B > \underline{v}-\overline{v}. \end{cases} \end{aligned}$$

Letting $\Delta_a = \underline{v} - \overline{v}$ and $\Delta_b = \overline{v} - \underline{v}$, firm 1's gross profit is

$$\pi_1(p_1^A, p_1^B, p_2^A, p_2^B)$$
$$= \begin{cases} \Big(F(p_2^A - p_1^A) + F(p_2^A - p_1^A)\Big)(p_1^A - c) & \text{if} \quad p_1^A - p_1^B \leq \Delta_a \wedge p_2^A - p_2^B \leq \Delta_a, \\ \Big(F(p_2^A - p_1^A) + F(\Delta_a + p_2^B - p_1^A)\Big)(p_1^A - c) & \text{if} \quad p_1^A - p_1^B \leq \Delta_a \wedge \Delta_a < p_2^A - p_2^B \leq \Delta_b, \\ \Big(F(\Delta_b + p_2^B - p_1^A) + F(\Delta_a + p_2^B - p_1^A)\Big)(p_1^A - c) & \text{if} \quad p_1^A - p_1^B \leq \Delta_a \wedge p_2^A - p_2^B > \Delta_b, \\ F(p_2^A - p_1^A)(p_1^A - c) + F(\Delta_b + p_2^A - p_1^B)(p_1^B - c) & \text{if} \quad \Delta_a < p_1^A - p_1^B \leq \Delta_b \wedge p_2^A - p_2^B \leq \Delta_a, \\ F(p_2^A - p_1^A)(p_1^A - c) + F(p_2^B - p_1^B)(p_1^B - c) & \text{if} \quad \Delta_a < p_1^A - p_1^B \leq \Delta_b \wedge \Delta_a < p_2^A - p_2^B \leq \Delta_b, \\ F(\Delta_b + p_2^B - p_1^A)(p_1^A - c) + F(p_2^B - p_1^B)(p_1^B - c) & \text{if} \quad \Delta_a < p_1^A - p_1^B \leq \Delta_b \wedge p_2^A - p_2^B > \Delta_b, \\ \Big(F(\Delta_a + p_2^A - p_1^B) + F(\Delta_b + p_2^A - p_1^B)\Big)(p_1^B - c) & \text{if} \quad p_1^A - p_1^B > \Delta_b \wedge p_2^A - p_2^B \leq \Delta_a, \\ \Big(F(\Delta_a + p_2^A - p_1^B) + F(p_2^B - p_1^B)\Big)(p_1^B - c) & \text{if} \quad p_1^A - p_1^B > \Delta_b \wedge \Delta_a < p_2^A - p_2^B \leq \Delta_b, \\ \Big(F(p_2^B - p_1^B) + F(p_2^B - p_1^B)\Big)(p_1^B - c) & \text{if} \quad p_1^A - p_1^B > \Delta_b \wedge p_2^A - p_2^B > \Delta_b. \end{cases}$$

Firm 2's gross profit is analogous.

To compute the NE of the pricing subgame, we proceed as follows for each of the possible constraints on the prices charged:

1. Assume that the constraint is satisfied and solve the resulting system of FOCs.

2. Verify that the solution satisfies the constraint and discard it otherwise. Note that the constraint, $p_n^A - p_n^B \leq \Delta_a$ ($p_n^A - p_n^B > \Delta_b$), is trivially satisfied because firm n is always free to set $p_n^B = \infty$ ($p_n^A = \infty$).

3. Verify that neither firm has a profitable unilateral deviation and discard the solution otherwise.

The above checks always rule out all but (at most) one solution. Hence, if it exists, the NE of the pricing subgame is unique.

Turning from price competition to product offerings, our computations show that for given parameter values the game with product-specific prices generally has at least as many SPEs as the original game.[8] For example, if $\underline{v} = -0.96$, $\overline{v} = 0.04$, $\beta = 0.5$, and $f = 0.05$, then both games have the same SPEs (see Section 4). On the other hand, if $\underline{v} = -0.92$, $\overline{v} = 0.08$, $\beta = 0.5$, and $f = 0.05$, then the original game has the following SPEs: one where one firm offers A and the other offers B, and one where both firms randomize between offering A and B with probabilities 0.5 and 0.5. The game with product-specific prices has a number of additional SPEs: one where both firms offer both A and B; one where one firm randomizes between offering GP and both A and B with probabilities 0.94 and 0.06, and the other randomizes between offering A and both A and B with probabilities 0.01 and 0.99; one where one firm randomizes between offering GP and both A and B with probabilities 0.94 and 0.06, and the other randomizes between offering B and both A and B with probabilities 0.01 and 0.99; one where one firm randomizes between offering GP, A, and both A and B with probabilities 0.77, 0.03, and 0.20, and the other randomizes between offering GP, B, and both A and B with probabilities 0.77, 0.03, and 0.20; and one where both firms randomizes between offering GP, A, B, and both A and B with probabilities 0.87, 0.02, 0.02, and 0.10. In both games, however, the unique Pareto-undominated equilibrium is market segmentation with niche firms.

For given parameter values the Pareto-undominated equilibrium is usually the same in both games, so that Figures 2-5 change very slightly.[9] In fact, the sole change is that for some parameter values market segmentation occurs via niche firms in the game with product-specific prices but via full-line firms in the original game. In contrast, the same parameter values lead to a general purpose equilibrium in both games. Overall, our conclusions from the main text continue to hold in the game with

[8] There are five exceptions in $25^2 \times (3+3+4+6) = 10000$ parameterizations. These are given by $\underline{v} \in \{-0.8, -0.76, -0.72, -0.68\}$, $\overline{v} = 0.44$, $\beta = 4$, and $f = 0.3$ and $\underline{v} = -1$, $\overline{v} = 0.6$, $\beta = 4$, and $f = 0.4$.

[9] The corresponding figures for the game with product-specific prices are available in the Online Appendix at the *Journal's* website.

product-specific prices.

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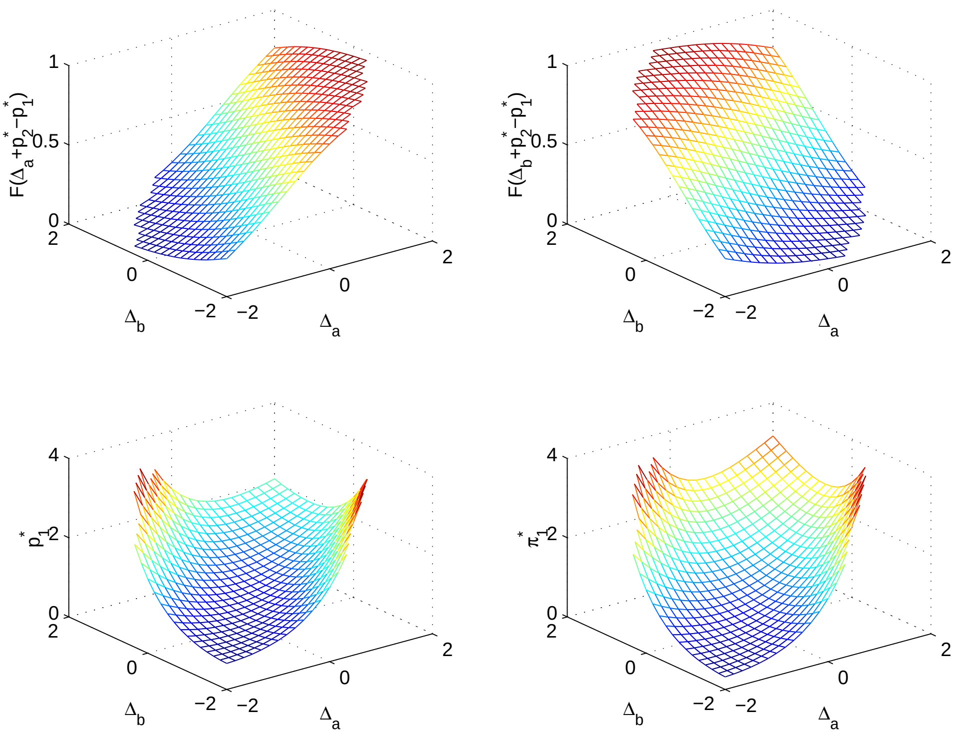

Figure 1: Equilibrium market shares of firm 1 in segments a and b, $F(\Delta_a + p_2^*(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$ and $F(\Delta_b + p_2^*(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$, equilibrium price of firm 1, $p_1^*(\Delta_a, \Delta_b)$, and equilibrium profit of firm 1, $\pi_1^*(\Delta_a, \Delta_b)$, for $\beta = 0.5$ and $c = 0$.

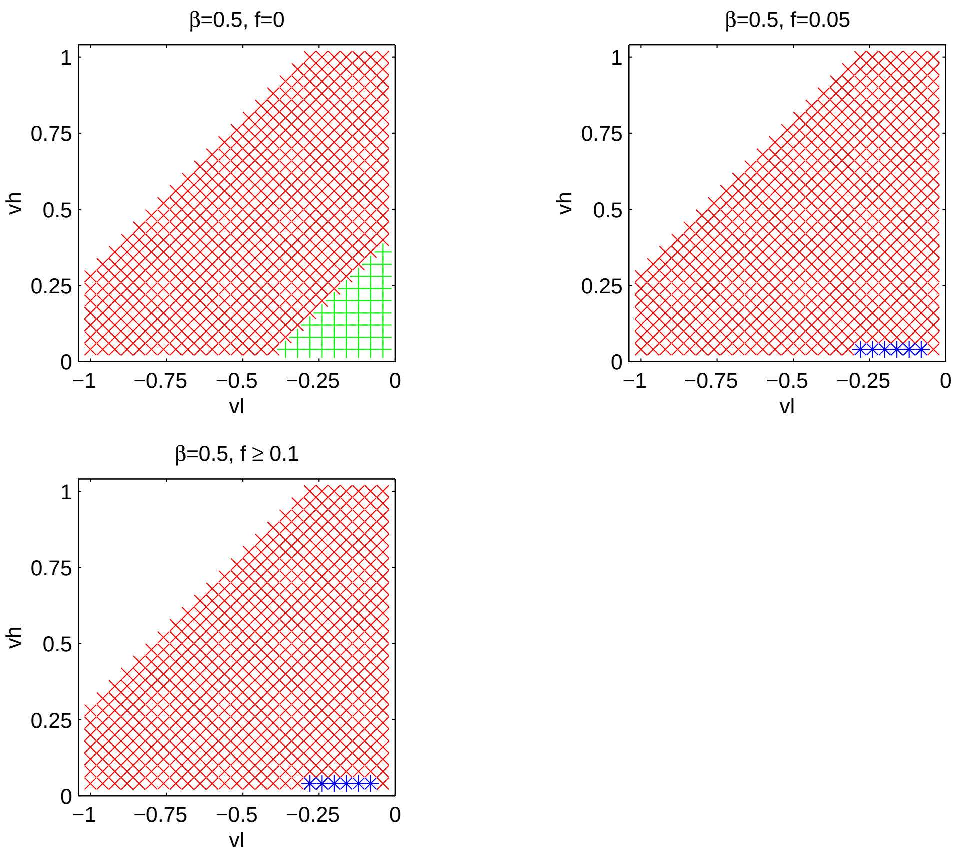

Figure 2: Pareto-undominated equilibria for $\beta = 0.5$, $f \in \{0, 0.05\}$, and $f \geq 0.1$. (GP, GP) is denoted by a blue star, (A, B) (or (B, A)) by a red x-mark, and (AB, AB) by a green plus.

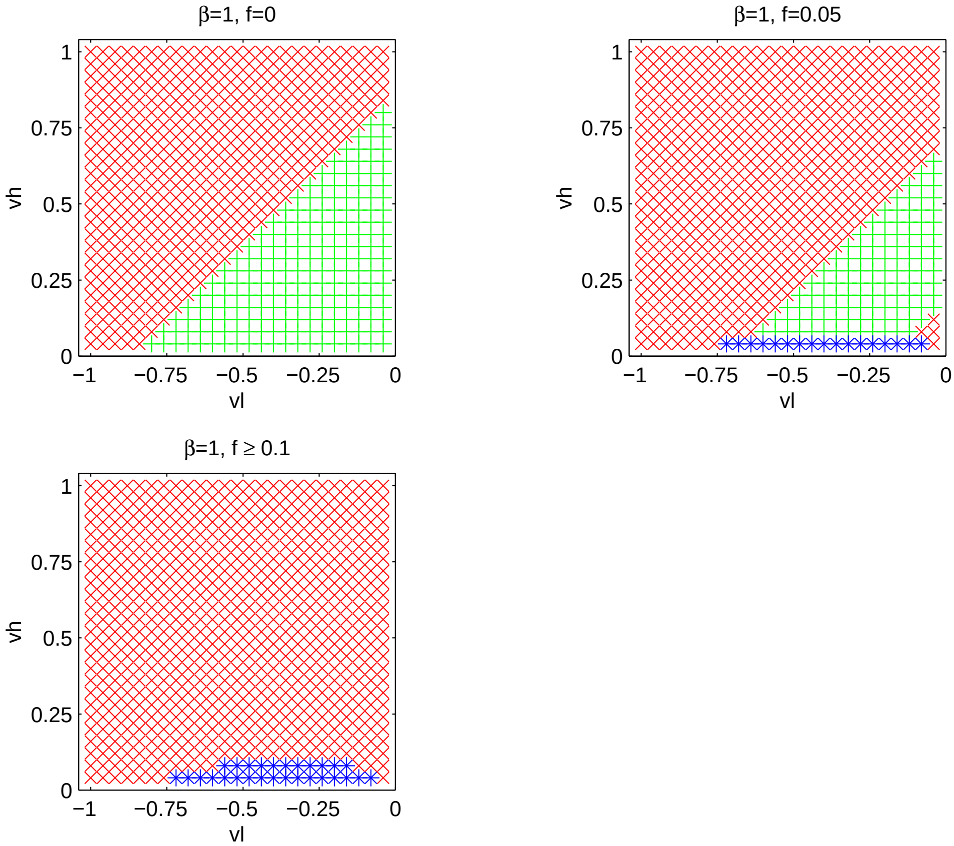

Figure 3: Pareto-undominated equilibria for $\beta = 1$, $f \in \{0, 0.05\}$, and $f \geq 0.1$. (GP, GP) is denoted by a blue star, (A, B) (or (B, A)) by a red x-mark, and (AB, AB) by a green plus.

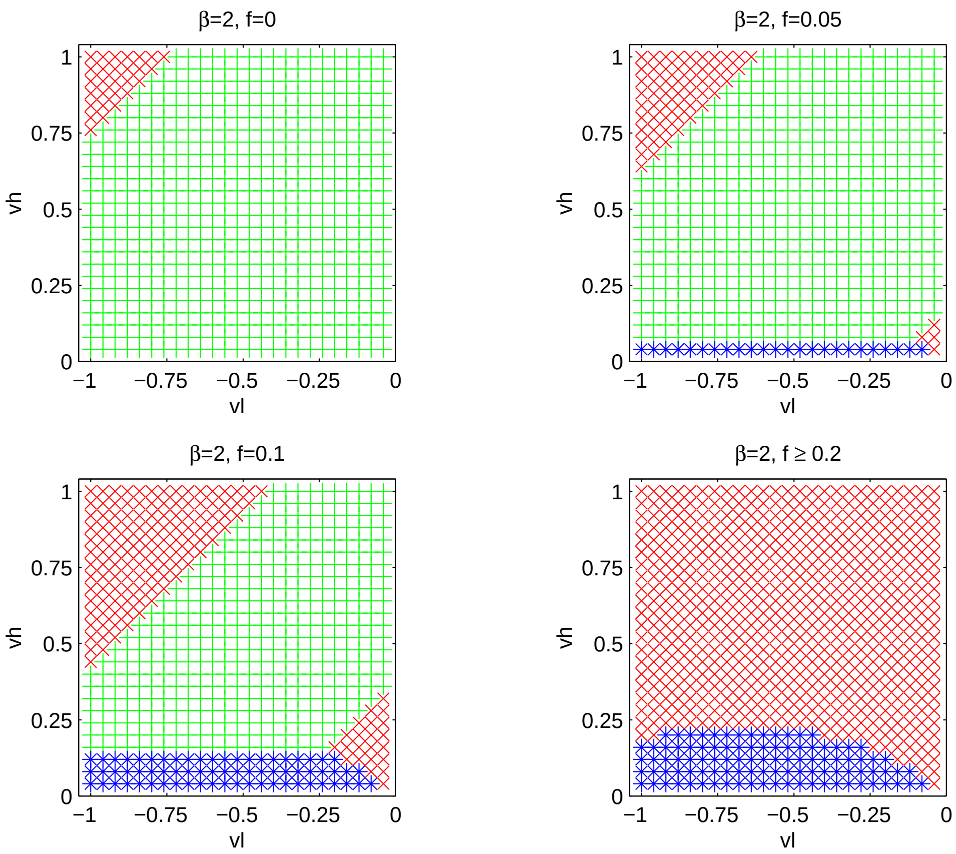

Figure 4: Pareto-undominated equilibria for $\beta = 2$, $f \in \{0, 0.05, 0.1\}$, and $f \geq 0.2$. (GP, GP) is denoted by a blue star, (A, B) (or (B, A)) by a red x-mark, and (AB, AB) by a green plus.

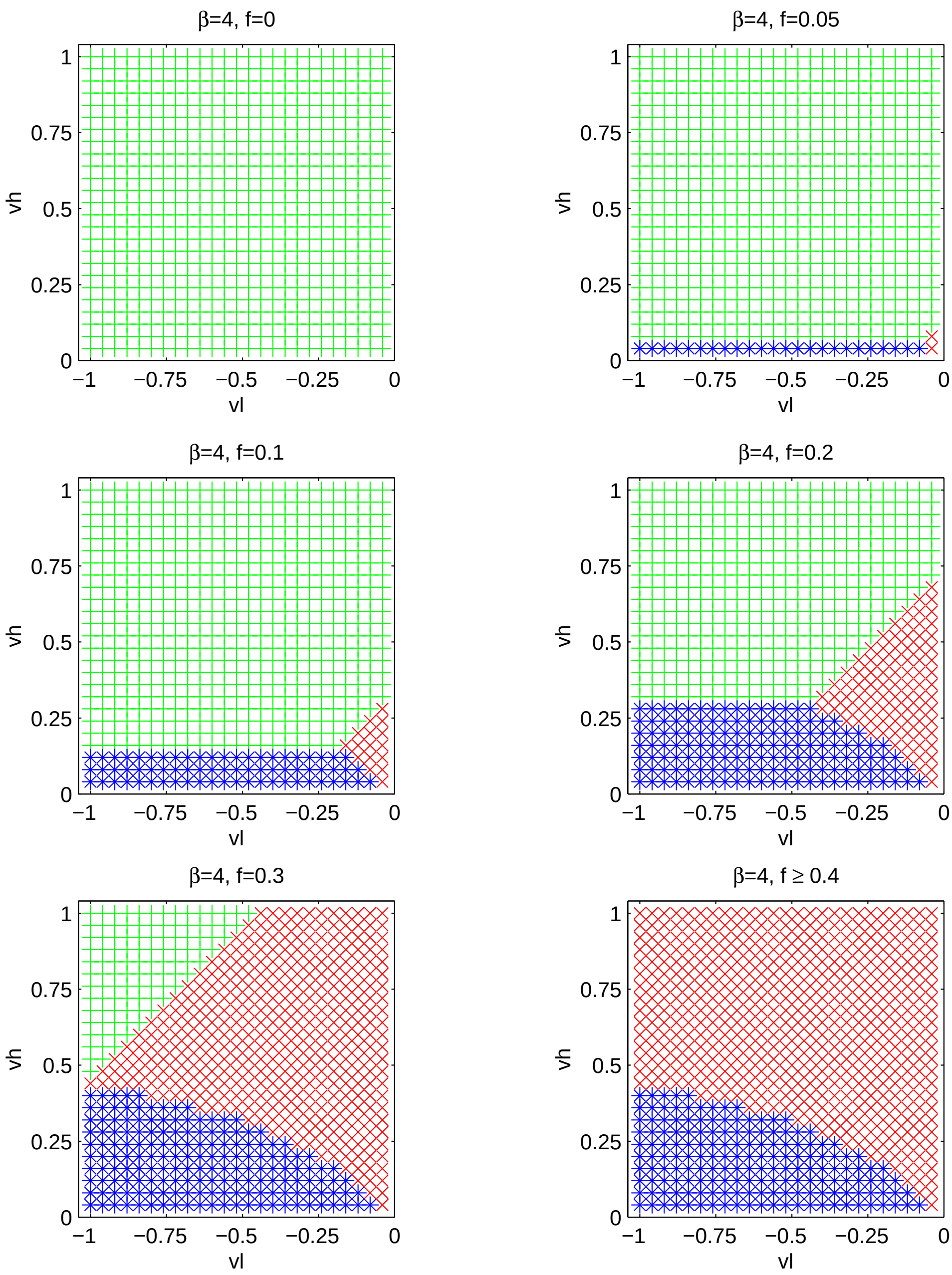

Figure 5: Pareto-undominated equilibria for $\beta = 0.5$, $f \in \{0, 0.05, 0.1, 0.2, 0.3\}$, and $f \geq 0.4$. (GP, GP) is denoted by a blue star, (A, B) (or (B, A)) by a red x-mark, and (AB, AB) by a green plus.